The Demand for Currency versus Debitable Accounts: a Reconsideration*

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Abstract

Modeling the demand for currency and deposits is a primary concern for central banks. Within a wide range of academic contributions, payment choice models based on transaction sizes (TS models) have been recently promoted. However, TS models induce strong predictions about the use of payment instruments. Especially, all equal-sized transactions should be paid with the same payment instrument. Hence, for each individual, one should observe strict domains of transaction for every payment instruments. Using micro-level payment data from a representative sample of the French population, we show that TS models are bad at replicating individual and aggregate payment patterns. First, we show that the predictions of TS models are not empirically validated on an individual level. Second, we develop and test three models to explain the observed aggregate payment patterns. The first two models are aggregate versions of TS models and the third one is an alternative model based on a payment decision rule depending on cash holding (CH model). We find that the third model gives predictions between 2 and 6 times more precise than the first two with notably less demanding information on individuals.

Key Words: Demand for money, Payment Instruments.
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1 Introduction

For several decades, monetary economists have attached a considerable interest in modeling demand for currency and deposits.[1] Within a wide range of contributions, two distinct lines of analysis have been proposed.

The first one, represented by Santomero (1974, 1979), Santomero and Seater (1996) and their references, have developed formal models in which demand for currency and deposits is a direct function of rates of return and fixed costs of transaction. Although important, these models have never been estimated since their complexity thwart all empirical application (Folkertsma and Hebbink, 1998). The second line of analysis promoted by Whitesell (1989, 1992) relies on simpler and more tractable models. Conversely to Santomero and Seater for whom transaction costs are assumed independent of the amounts paid, Whitesell shows that when the transaction costs for each payment instrument differ and when this difference depends on the transaction size, the demand for currency and deposits may be considerably affected. A central implication of these models is that all equal-sized transactions will be paid with the same payment instrument and hence, each payment instrument will be used exclusively on a particular domain of transaction: cash for transactions with size below a given threshold and alternative payment instruments (debitable account) for transactions with size above the same threshold. Hence, these models are based on transaction sizes and are called hereafter "TS models".

The second line of analysis has been extended by Shy and Tarkka (2002) in the case of the electronic money and has also been tested in empirical studies. Mot and Cramer (1992), Boeschoten (1992), Hayashi and Klee (2003), Bounie and François (2006) as well as Klee (2008) have confirmed for instance that the probability to use a given payment instrument was highly influenced by the price of the transaction. However, even though the transaction size can

[1] Debitable accounts and deposits are not differentiated in this article.

be considered as a statistically significant variable in explaining the choice of payment instruments and hence the demand for money and deposits, the ability of TS models to replicate individual and aggregate payment patterns have, to the best of our knowledge, never been tested. The main objective of this paper is to determine to what extent TS models can contribute at an individual level to explain consumers' payment behaviour and can provide at an aggregate level a good prediction of the demand for currency and deposits.

In doing so, we use payment data from a representative sample of the French population. First, we find that 32.3% of the people who hold cash and an alternative payment instrument do not behave as TS models predict. Indeed, for those individuals, we do not observe strict domains of transaction for the payment instruments but rather overlapping transaction domains. In order to answer the question at an aggregate level, we test three models for their ability to replicate the observed aggregated data. The first two models are TS models with two different aggregation rules. The first one is naive and considers the whole population as a representative agent satisfying the assumptions of the TS models. As expected, this model performs very poorly. The second model considers a representative agent with a probabilistic threshold *à la* Whitesell. The probability distribution of the threshold is given by some actual realistic thresholds observed in the data. This latter model performs much better but is very demanding in terms of information. The third model we develop is an alternative model of payment choice by a representative agent based on a simple payment decision rule, based on cash holding ("CH model"). We find that the CH model best fits the data of the observed aggregate payments. More precisely, we show that the CH model gives predictions between 2 and 6 times better than aggregate TS models with notably less requiring information basis on individual payment patterns. Indeed, the CH model simply relies on three sorts of information: the distributions of prices and cash withdrawals in the economy and the value of a threshold below which the representative agent goes back to withdraw cash. The CH model can be considered as a more efficient and less complex representation of the way a set of individuals pays.

The remainder of the paper is structured as follows. In the second section, we present the literature related to TS models and test their performance at the individual level. We show that our micro-level data partly invalidate TS models. In the third section, we propose three models of aggregate payment pattern. Two are derived from TS models, the last one, the CH model, is alternative. We show that the last model fits best the observed payment data. In the fourth section, we discuss the possibility of extending our study for more than two payment instruments. The last section concludes.

2 The TS models of individual decisions

2.1 Theory

The literature on transaction size based models (TS models) goes back to the transaction demand for money *à la* Baumol (1952) and Tobin (1956). In these studies, a cost-minimizing consumer has to decide the optimal stock of cash to be held for transaction purposes given the cost of a withdrawal (a fixed fee per withdrawal) and the interest earnings foregone on money holding. Extending this approach to several payment instruments, Whitesell (1989, 1992) explicitly assumes that the consumer's problem during a purchase merely consists in choosing a payment instrument which minimizes the holding and transaction costs. In Whitesell (1989), for instance, an individual is assumed to make purchases of various prices P, with $0 < P < \infty$. Transactions are supposed to be uniformly distributed over a continuous unit period. Then, if $F(P)$ represents the value of spending on all transactions of price P, the expected total spending is given by $Y = \int_0^\infty F(P)dP$. The individual has the possibility to pay with cash or with an alternative payment instrument. Each transaction carried out with the alternative payment instrument is attended by some costs. Let us note u the fixed cost of using the alternative payment instrument for a transaction of size P and $v.P$ the variable cost related to P. The total cost of a transaction of size P is then $C = u + v.P$. On the contrary, paying cash is attended only by a variable cost $w.P$ for a purchase of size P. This cost is due to interest earnings foregone on money holding. Then, the alternative payment instrument will not be used for small value transactions because of u and, if the parameters are well chosen, cash will be only used for small value payments, for which the interest earnings foregone will be minimal.

2.2 An empirical verification of the TS models

In this section, we test the performance of the TS models. In so doing, we use data collected on payment patterns of French representative individuals. First, we present the methodology of the survey and the data collected. Second, we perform the test.

2.2.1 Methodology of the survey

We administrated a survey from March to May 2005 on a representative sample of 1,447 French individuals of 18 years and older.[2] The survey was realized in two steps. First, we surveyed individuals, during face-to-face interviews, in order to collect information on their payment instrument equipment, their cash management, etc. Second, we asked each respondent to keep a diary in which they reported all information related to purchases on a daily basis, for eight days.[3] A purchase is characterized by several pieces of information such as the amount to be paid (size of transaction) and the type of payment instrument used to settle the purchase (cash, check, payment card, etc.). As a result, we were able to calculate, for each respondent, the smallest and largest transactions paid with each payment instrument.

[2]Respondents were never participated to any survey before.

[3]Professional expenses and bill payments were excluded from diaries.

In other words, we can check the existence of transaction domains for each individual.

Globally, we observe that over 1,447 respondents, 1,392 individuals have completed the diary. Overall, we have 16,692 transactions available containing all the information we need for this study.[4] A description of the distribution of purchases paid cash or with an alternative payment instrument are plotted in Figure 1.[5] A few comments may be made. First, the major part of the transactions are small value purchases (below 20 euros). Second, most of the transactions are paid cash (64%; see Appendix A for more descriptive statistics). Third, as can be seen in Figure 2, and as already noticed in the Introduction, the amount of a transaction is statistically important to predict if it will be paid in cash or using the alternative payment instrument.

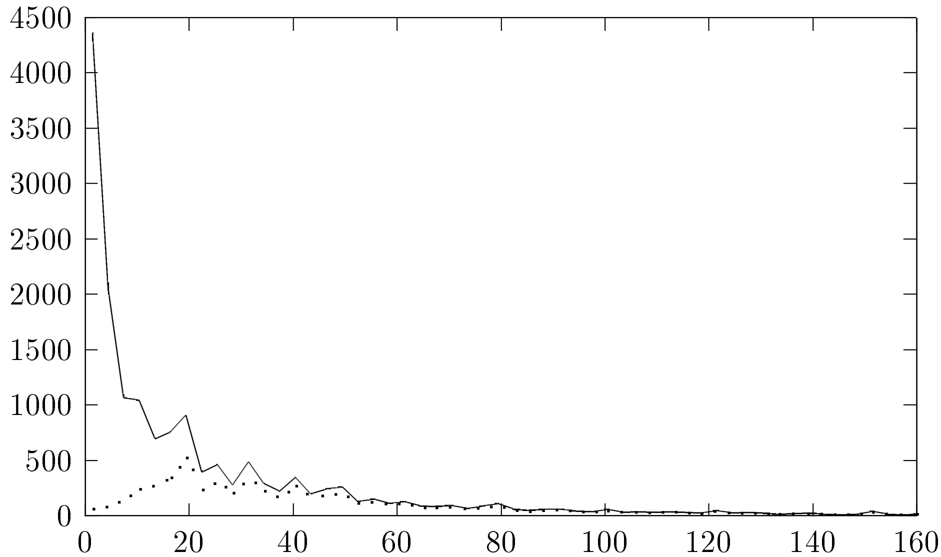

Figure 1: Total number of purchases (solid line) and number of purchases not paid in cash (dotted line) in function of the price.

[4]6 individuals have no access to any payment instrument but cash. We did not take them into account.

[5]In all the figures we show, the data are summed for price classes of 3 euros thickness.

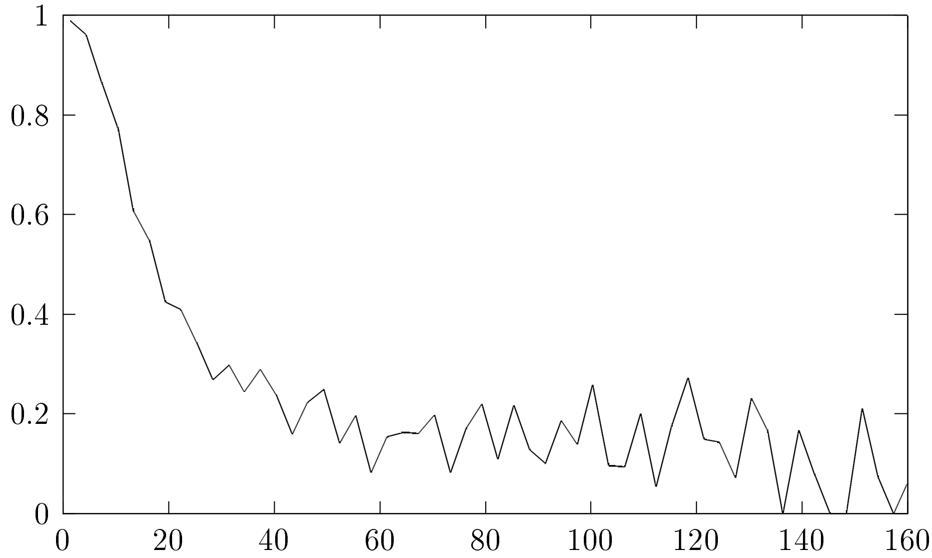

Figure 2: Share of the purchases paid in cash (below the solid line) and with an alternative payment instruments (above the solid line) in function of the price.

2.2.2 Empirical verification

Now, let us try to test the TS models results at an individual level for people who have cash and an alternative payment instrument. As in Whitesell (1989), these individuals will use cash or the alternative payment instrument respectively for small and large transactions respectively. More precisely, for any individual, if P_{cash}^{max} is the maximum value of a transaction paid with cash and $P_{alternative}^{min}$ is the minimum value of a transaction paid with an alternative payment instrument, then we should always observe:

$$P_{cash}^{max} \leq P_{alternative}^{min}. \tag{1}$$

A first analysis of the payment data shows that some individuals satisfy Equation (1). At the top of Figure 3, we reproduced the payment patterns of such an individual who holds cash and an alternative payment instrument. We can see two strict domains of transaction: the largest transaction paid cash (around 15 euros) is strictly lower than the smallest purchase paid with the alternative payment instrument (around 20 euros).

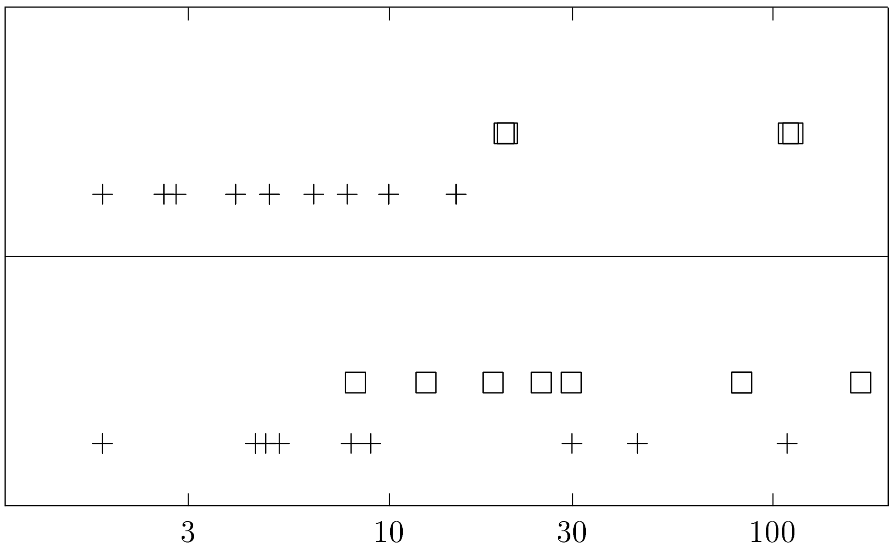

Figure 3: Examples of use of cash (+) and alternative payment instruments (□) in function of the price for two individuals.

However, surprisingly, we also find individuals who do not behave as in TS models. For instance, we reproduced the payment pattern of such a typical individual at the bottom of Figure 3. We can see two overlapping domains of transaction. Indeed, we observe that the largest transaction paid cash, around 120 euros, is strictly larger than the smallest purchase paid with the alternative payment instrument (around 7 euros). As a result, such an individual does not conform to Equation (1) and hence, does not behave as TS models predict.

A more global analysis of individual payment patterns shows that TS models do not account well for the way people pay. In fact, we observe that 32.3% of the people who use cash and an alternative payment instrument do not satisfy Equation (1).

In the present section, we have shown that TS models poorly perform at the individual level. However, it would be possible that TS models remain valid at an aggregate level, *i.e.* when the whole population is considered rather than each individual. The next section shows that TS models are not good at replicating the data at an aggregate level either.

3 Three models of aggregate decision

In this section, the purpose is to test several models at an aggregate level. In order to do that, we need to specify how individual decisions become aggregate decisions. We propose two ways to do that for TS models. We also propose a third model that considers a representative agent that makes his decision differently from what TS models propose.

3.1 The naive TS model

The most natural way to aggregate individual decisions into a population decision is to consider a representative agent that behaves like a single individual. This aggregation is of course a bit naive but it is a good benchmark to start our study of the use of cash at an aggregate level.

Then, let us consider a representative agent who pays cash if and only if the price of the good he purchases is smaller than a threshold L and pays with the alternative payment instrument if and only if the price of the good he purchases is greater than L. Formally, if $\widehat{F_L^{th}}(p)$ is the theoretical probability of paying in cash for a purchase of price p, then

$$\widehat{F_L^{th}}(p) = \begin{cases} 1 \text{ if } p \leq L \\ 0 \text{ if } p > L \end{cases} .$$

3.2 The differentiated TS model

Let us now try to improve the naive way to aggregate individual decisions satisfying TS models predictions by extending the informational basis. We will call this aggregate decision model, the differentiated TS model. As we have seen in Section 2.2.2, many individuals do not satisfy the TS models approach. However, we can try to extract from the individuals' behavior some thresholds for which they violate the TS model implications as rarely as possible. Let us first give an example illustrated in Figure 4.

Let us consider an individual i that purchases three goods. The first one, good 1, is 5 euros worth and is paid cash, the second one, good 2, is 10 euros worth and is paid with the alternative payment instrument, the third one, good 3, is 15 euros worth and is paid cash. If we use the TS model approach as described in Section 2 with a threshold between 0 and 5 euros, 2 data cannot be explained, namely, theoretically goods 1 and 3 should be paid with the alternative payment instrument whereas, by assumption, they are paid cash. If we use a threshold between 5 and 10 euros, 1 data cannot be explained, namely, theoretically, good 3 should be paid with the alternative payment instrument whereas, by assumption, it is paid cash. If we use a threshold between 10 and 15 euros, 2 data cannot be explained, namely, good 2 should be paid cash and good 3 should be paid with the alternative payment instrument. If we use a threshold above 15 euros, 1 data cannot be explained, namely, good 2 should be paid with the alternative payment instrument. Then, the set of prices $S_i = [5, 10] \cup [15, \infty[$ can be interpreted as the values for which a threshold would make TS models the most acceptable. To avoid problems dealing with infinite or 0-value thresholds, we will consider the set $\mathcal{S}_i = [P_i^{min}, P_i^{max}] \cap S_i$ as the set of acceptable thresholds for individual i where P_i^{min} is the value of the cheapest good purchased by individual i and P_i^{max} is the value of the most expensive good purchased by individual i. We define $\overline{l_i} = \max\{l \in \mathcal{S}_i\}$ and $\underline{l_i} = \min\{l \in \mathcal{S}_i\}$. Then, $\overline{l_i}$ is the greatest value between P_i^{min} and P_i^{max} for which TS models make the minimum number of mistakes concerning the purchases of individual i. $\underline{l_i}$ is the smallest value between P_i^{min} and P_i^{max} for which TS models make the minimum number of mistakes concerning the purchases of individual i. We define $\overline{l} = \{\overline{l_1}, ..., \overline{l_{1386}}\}$ and $\underline{l} = \{\underline{l_1}, ..., \underline{l_{1386}}\}$ the sets of thresholds found for the individuals in the population we have data for. We show some descriptive statistics about the observed thresholds in Table 1, page 21.

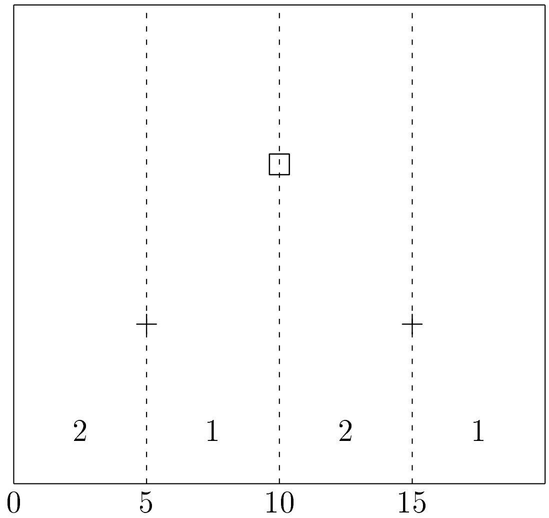

Figure 4: Examples of TS model errors for different thresholds when using cash (+) or an alternative payment instrument (□).

Instead of a representative agent having a constant threshold L as in the naive aggregation of TS models, we can now model a representative agent with a probabilistic threshold. When facing a price p, the representative agent draws by chance a threshold $l \in \bar{l}$ and pays cash if and only if $p \leq l$. The probability of drawing $l = l_i$ is given by $P(l = \bar{l_i}) = \dfrac{n_i}{\sum_j n_j}$ where n_i is the number of transactions made by individual i in the observed data. Computing this new aggregate decision rule gives the probability $\widetilde{F^{th}_{\bar{\mathbf{l}}}}(p)$ for each good worth p euros to be paid cash. Considering $\underline{l}$ instead of $\bar{l}$ in the previous decision rule gives $\widetilde{F^{th}_{\underline{\mathbf{l}}}}$.

3.3 The CH model

Let us now describe the CH model. Unlike the TS models, the CH model is not only based on the prices of the goods. More crucially, it depends on the amount of cash the agent is holding, hence it is more a cash holding model.

Let us consider an agent carrying m euros in cash. When facing a price p, the agent pays p euros in cash if and only if $m \geq p$. Else, if $m < p$, the agent pays p euros using the alternative payment instrument. Then, after each purchase, the agent has $m' = m - p$ euros (if $m \geq p$) or $m' = m$ euros (if $m < p$) left in cash. Then, the agent needs to make a decision concerning his cash holding level. Following the "bottom inventory" assumption of Boeschoten (1992, p: 43), we assume that the agent withdraws some cash if and only if his cash holding is smaller than a threshold l, *i.e.* if $m' \leq l$. If the agent decides to withdraw some cash, the amount he withdraws, w, is drawn form the statistical distribution, $W(.)$, of the observed withdrawals in the economy.

A complete axiomatization of this model is given in Bounie and Houy (2007). Simplifying a bit, we list here some crucial assumptions behind this decision rule.

1. The individual is always interested in buying the good. The surplus the individual gets from the good is superior to the cost of the transaction to buy it.

2. When the individual has the choice between cash and an alternative payment instrument, he prefers to use cash. This behavior can be explained by the cost of cash holding which mainly depends on the interest earnings foregone, the risk of loss and theft as well as the inflation cost (Baumol, 1952; Tobin, 1956).

3. The individual goes to an Automated Teller Machine (ATM) when his cash holding is below a given threshold. Besides what has been said before, an individual needs to hold a minimal amount of cash for several reasons. First, cash is the only legal tender in the economy. Second, there is no alternative payment instrument different from cash available to settle low value transactions since payment cards and checks are not always accepted below a threshold by some retailers. Third, and more generally, alternative payment instruments such as payment cards are

not necessarily accepted in the whole merchant places. Four, cash can be required to face specific transactions such as those at vending machines.

3.4 Simulations and results

3.4.1 Method

In order to gauge how well the three models replicate the data, we use numerical simulations.

Let us consider a representative agent. At each period of time, the agent is facing a price p. p is drawn by chance from the distribution $P(.)$. Then, the agent makes his decision about the payment instrument according to one of the decision rules described above. If we test the naive TS model with a threshold L, the agent pays in cash if and only if $p \leq L$, else, he pays with the alternative payment instrument. If we test the differentiated TS model with threshold distribution, $\bar{l}$, the agent draws by chance a threshold in $\bar{l}$ as described above and pays in cash if and only if p is smaller than this threshold. If we test the differentiated TS model with threshold distribution, $\underline{l}$, the agent draws by chance a threshold in $\underline{l}$ as described above and pays in cash if and only if p is smaller than this threshold. If we test the CH model, the agent pays in cash if and only if he holds enough cash and in a second step makes his decision about withdrawing cash as described above.

From our data set, we know the distribution of the prices purchased in the economy, $P(.)$. We also know the distribution of withdrawals, $W(.)$, needed for the CH model.[6]

Let us give a sequence as an example for the naive TS model with $L = 25$. $L = 25$ means that the individual pays in cash if and only if the price is smaller than 25. Assume that, first, the agent faces a buying decision for a good worth 75 euros. Since this good is more expensive than $L = 25$ euros,

[6]For a statistical description of transactions and withdrawals, see Tables 2 and 3 in Appendices A and B respectively.

it is paid with the alternative payment instrument by the agent. Assume, that, in a second period, the agent faces a buying decision for a good worth 30 euros. Since this good is more expensive than $L = 25$ euros, it is paid with the alternative payment instrument by the agent. Assume that, in a third period, the agent faces a buying decision for a good worth 21 euros. Since this good is less expensive than $L = 25$ euros, it is paid cash by the agent. If we were stopping our simulation here, we would have, $\widehat{F_{25}^{th}}(21) = 1$, $\widehat{F_{25}^{th}}(30) = 0$ and $\widehat{F_{25}^{th}}(75) = 0$. Obviously, it is straightforward to check that with an infinite periods simulation, we have $\widehat{F_{L}^{th}}(p) = 1$ if $p \leq L$ and $\widehat{F_{L}^{th}}(p) = 0$ if $p > L$.

Let us give the same sequence as an example for the differentiated TS model with $\bar{l}$. Assume that in the first period, when the price faced is 75 euros, the agent draws by chance a threshold $l = 80$. Then, the agent pays in cash since $l \geq 75$. Assume that in the second period, when the price faced is 30 euros, the agent draws by chance a threshold $l = 20$. The agent pays with the alternative payment instrument since $l < 20$. Assume that in the third period, when the price faced is 21 euros, the agent draws by chance a threshold $l = 20$. The agent pays with the alternative payment instrument since $l < 21$. If we were stopping our simulation here, we would have, $\widetilde{F_{\bar{l}}^{th}}(21) = 0$, $\widetilde{F_{\bar{l}}^{th}}(30) = 0$ and $\widetilde{F_{\bar{l}}^{th}}(75) = 1$. Notice that the representative agent satisfying the differentiated TS model is not necessarily satisfying the predictions of TS models for individuals. Indeed, since the threshold can vary, it is possible that a purchase be paid in cash whereas a cheaper one would be paid with the alternative payment instrument.

Let us give the same sequence as an example for the CH model with $l = 5$. $l = 5$ means that the individual goes to withdraw cash when the amount of cash he holds is below 5 euros. Moreover, let us assume an agent holding 100 euros in cash at the beginning of times.[7] When the representative agent

[7]Notice that this information is irrelevant for TS models and it is also irrelevant when the simulation is considering an arbitrarily large number of periods.

is facing the 75 euros good, since the agent can pay with cash, so does he. Then, he has 25 euros remaining in cash. Since the threshold l is not reached from above, the individual does not withdraw any cash. When facing the second good, worth 30 euros, since the agent cannot pay with cash, he pays with the alternative payment instrument. He has still 25 euros in cash and for the same reasons as above, does not withdraw any cash. When facing the third purchase opportunity, worth 21 euros, since the agent can pay in cash, so does he. He has now 4 euros in cash remaining and since his cash stock is below l, he goes to an ATM or to his bank to refill. If he withdraws, say 20 euros, the model goes on with the agent having 24 euros in cash.

Iterating this algorithm an infinite number of times with a representative agent satisfying the naive TS model, we can compute $\widehat{F_L^{th}}(p)$, depending on the threshold chosen. We can also compute $\widetilde{F_{\bar{l}}^{th}}(p)$ (resp. $\widetilde{F_{\underline{l}}^{th}}(p)$), the theoretical frequency with which the representative agent satisfying the differentiated TS model with thresholds distribution $\bar{l}$ (resp. $\underline{l}$) pays in cash at each price p. Finally, we can compute $F_l^{th}(p)$ the theoretical frequency with which the representative agent satisfying the CH model with threshold l pays in cash for each price p.

The purpose is now to find a measure to show how well each of these theoretical models replicate the data. The distance we use is as follows. Let us denote by $F^{obs}(p)$ the observed share of purchases worth p euros paid cash. Let us denote by $f^{th}(p)$, the theoretical share of purchases worth p euros paid cash when one of the models described above is considered ($f^{th}(.) \in \{\widehat{F_L^{th}}(.), \widetilde{F_{\bar{l}}^{th}}(.), \widetilde{F_{\underline{l}}^{th}}(.), F_l^{th}(.)\}$ depending on the model we test). The distance $D(F^{obs}, f^{th})$ is given by

$$D(F^{obs}, f^{th}) = \sum_{p \in [0,\infty[} \frac{n(p)}{N} \mid F^{obs}(p) - f^{th}(p) \mid . \qquad (2)$$

where $n(p)$ is the number of purchases worth p in the data and $N = \sum_p n(p)$ is the total number of purchases observed. Then, this distance sums the differences between F^{obs} and f^{th} for each price p, each difference being weighted by the frequency $n(p)/N$ of purchases of price p. This weight seems quite natural since an error when many transactions are concerned should be more crucial than when only a few transactions are concerned. As an example, if one predicts that the individual will pay cash 95% of the purchases between 3 and 6 euros whereas the actual observed percentage is 96%, the error is more important than predicting that 1% of the purchases between 156 and 159 euros will be paid cash whereas the actual observed percentage is 0%. Indeed, almost 1/8 of all the purchases are between 3 and 6 euros whereas about 1/3000 of the purchases are between 156 and 159 euros. Some more remarks about the distance used are given in the Conclusion.

3.4.2 Results

Let us now give the results. For the naive aggregation of the TS model, the observed data show that more than 50% of the purchases are paid cash for prices less than 18 euros and paid with an alternative payment instrument for prices above 18 euros. Then, the minimum distance $D(F^{obs}, \widehat{F_L^{th}})$ is obtained for $L = 18$. More generally, the distance $D(F^{obs}, \widehat{F_L^{th}})$ in function of the threshold L is shown in Figure 5. We can interpret Figure 5 as follows. For small values of L, the agent often pays with the alternative payment instrument. At the limit $L \to 0$, the agent never pays cash. The observed probability that the agent pays cash is approximately 64%,[8] hence the difference between the observed data and the theoretical ones is approximately 64%. On the contrary, for great values of L, the agent often pays with cash. At the limit $L \to \infty$, the agent never pays with his alternative payment instrument. The observed probability that the agent pays with his alternative payment instrument is approximately 36%, hence the difference between the observed data and the theoretical ones goes to approximately 36%.[9] In between those values, the representative agent's theoretical behav-

[8]See Table 2.

[9]See Table 2.

ior gets closer to what is observed. The minimum error is reached at the value $L = 18$ where we have $D(F^{obs}, \widehat{F^{th}_{18}}) = 17.1\%$. In other words, for a price p, the theoretical probability that an agent pays cash computed by the naive aggregation of the TS model model with $L = 18$, is on average 17.1% different from the observed probability. Figure 6 shows $F^{obs}(p)$ and $\widehat{F^{th}_{18}}(p)$ for values of p between 0 and 160 euros.

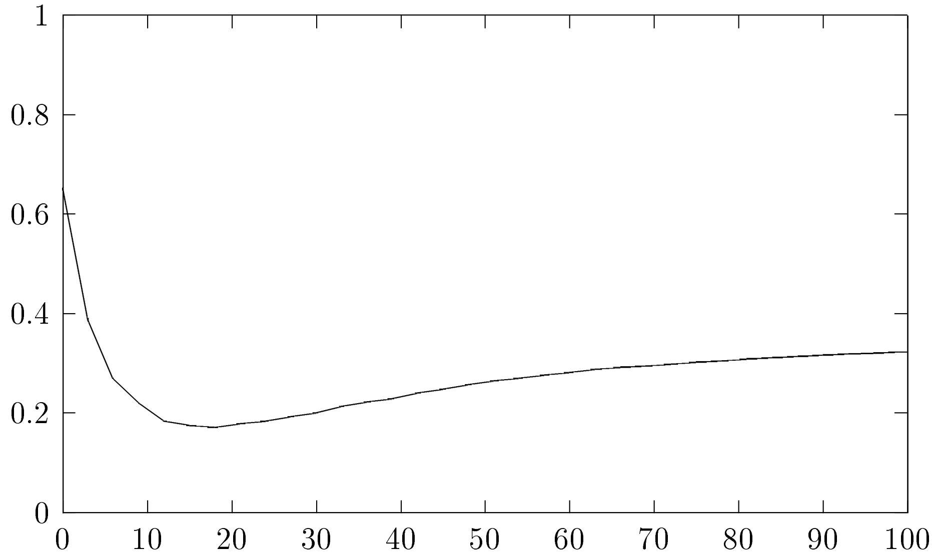

Figure 5: Distance $D(F^{obs}, \widehat{F^{th}_{L}})$ in function of the threshold L.

Not surprisingly, we will see that the naive aggregation of the TS model is the worse at replicating the observed aggregate payment. Indeed, as we have shown in Section 2.2.2, the TS model approach is not necessarily adapted to account for individual payment patterns. It would have been surprising that a naive aggregation of it perform well.

Let us now consider the differentiated aggregation of the TS model. $\widetilde{F^{th}_{\overline{\mathbf{l}}}}(p)$ and $\widetilde{F^{th}_{\underline{\mathbf{l}}}}(p)$, the theoretical frequencies obtained by the differentiated TS model are shown in Figure 7 together with $F^{obs}(p)$. Obviously, $\widetilde{F^{th}_{\overline{\mathbf{l}}}}(p)$ is above $\widetilde{F^{th}_{\underline{\mathbf{l}}}}(p)$ for each p. Indeed, by definition, $\overline{\mathbf{l}}_{\mathbf{i}} \geq \underline{\mathbf{l}}_{\mathbf{i}}$ for all individual i.

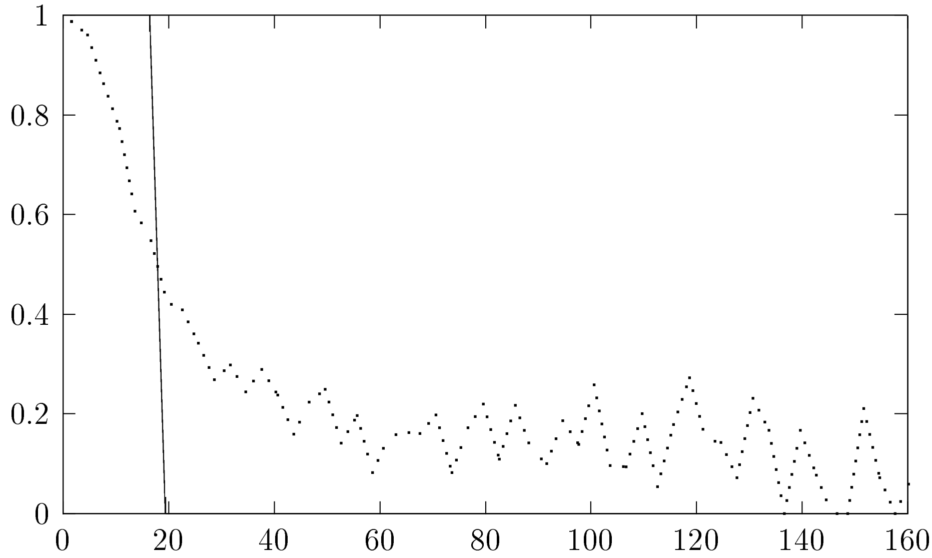

Figure 6: Observed F^{obs} (dotted line) and theoretical $\widehat{F_{18}^{th}}$ (solid line) frequencies of payment in cash in function of the price.

Then, the threshold above which purchases are made cash in the differentiated TS model is higher when $\bar{\mathbf{l}}_{\mathbf{i}}$ is considered rather than when $\underline{\mathbf{l}}_{\mathbf{i}}$ is. Hence, more purchases are made cash when using $\bar{\mathbf{l}}$ than when using $\underline{\mathbf{l}}$.

It is now straightforward to compute the distances, $D(F^{obs}, \widetilde{F_{\bar{\mathbf{l}}}^{th}})$ and $D(F^{obs}, \widetilde{F_{\underline{\mathbf{l}}}^{th}})$. We find $D(F^{obs}, \widetilde{F_{\bar{\mathbf{l}}}^{th}}) = 6.4\%$ and $D(F^{obs}, \widetilde{F_{\underline{\mathbf{l}}}^{th}}) = 10.8\%$. Hence, the differentiated TS model is 2 or 3 times better than the naive TS model at replicating the data. However, we will see that the CH model shows better predictive results.

We now comment on the results for the CH model. The distance $D(F^{obs}, F_l^{th})$ is shown in Figure 8 for different values of l. We can interpret Figure 8 as follows. According to the CH model, for small values of l, the agent goes rarely to the ATM and then carries little cash. Then, the representative agent often pays with an alternative payment instrument. At the limit $l \to 0$, the agent never pays cash. The observed probability that the agent pays cash is approximately 64%, hence the difference between the observed data and the

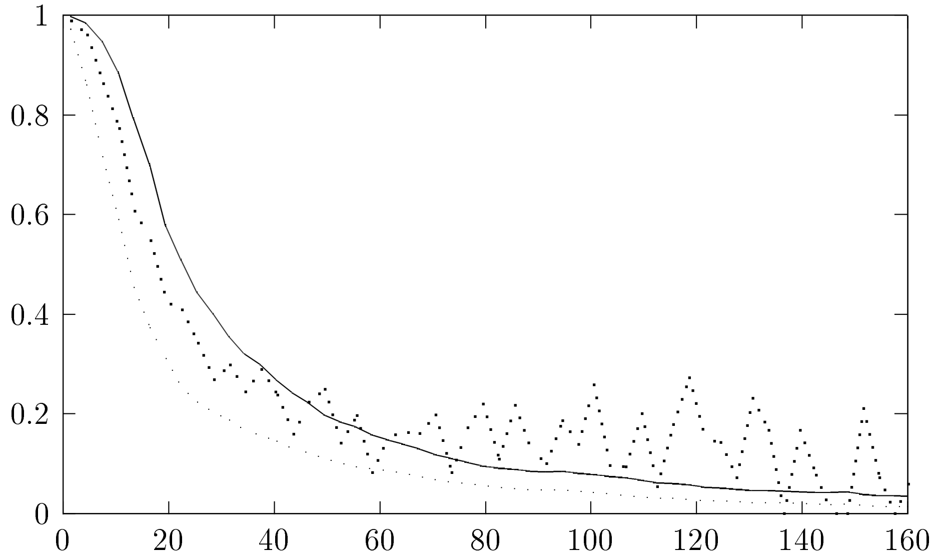

Figure 7: Observed F^{obs} (thick dotted line) and theoretical $\widetilde{F^{th}_{\overline{l}}}$ (plain line), $\widetilde{F^{th}_{\underline{l}}}$ (light dotted line) frequencies of payment in cash in function of the price.

theoretical ones is approximately 64%. On the contrary, for great values of l, the agent often withdraws cash. Then, he often pays with cash. At the limit $l \to \infty$, the agent never pays with any alternative payment instrument. The observed probability that the agent pays with his alternative payment instrument is approximately 36%, hence the difference between the observed data and the theoretical ones goes to approximately 36%. In between those values, the representative agent's behavior gets closer to what is observed.

We can note that the CH model fits the data with only an average error of less than 3.1% for a threshold $l = 4.1$. In other words, for a price p, the theoretical probability that a representative agent pays cash computed with the CH model with $l = 4.1$, is on average less than 3.1% different from the observed probability. This result suggests that the CH model has a very good predictive power. Indeed, it performs between 2 and 3 times better than the differentiated TS models.

The theoretical and observed probabilities to pay cash in function of the

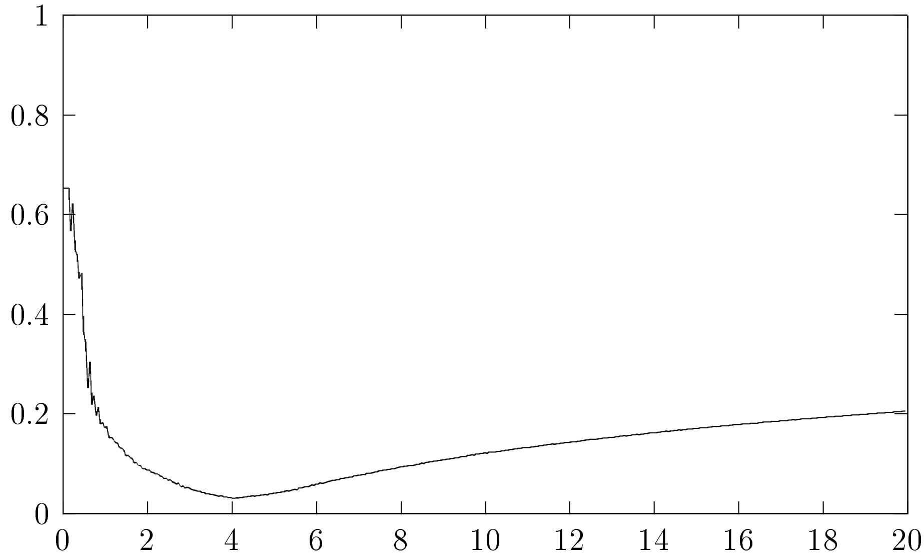

Figure 8: Distance $D(F^{obs}, F_l^{th})$ in function of the threshold, l.

price of the goods are shown in Figure 9 for the CH model. We can see that the CH model behaves particularly well for small prices (0-60 euros). The behavior for larger prices (80-140 euros) is not as good (the same remark could be done for the differentiated TS models). However, since, as can be seen in Figure 1, most of the transactions take place for small prices, the predictions of the CH models show a very good fit with the observed data.

We sum up our results in Table 1, page 21. Globally, we find that the CH model gives predictions from 2 to 6 times more precise than aggregate TS models and hence may be considered as more efficient than TS models.

4 Extension for three payment instruments

The previous analysis suggests that TS models do not replicate quite well the observed aggregate payment patterns of people who hold cash and an alternative payment instrument. However people may hold and use several alternative payment instruments related to deposits such as a check and a

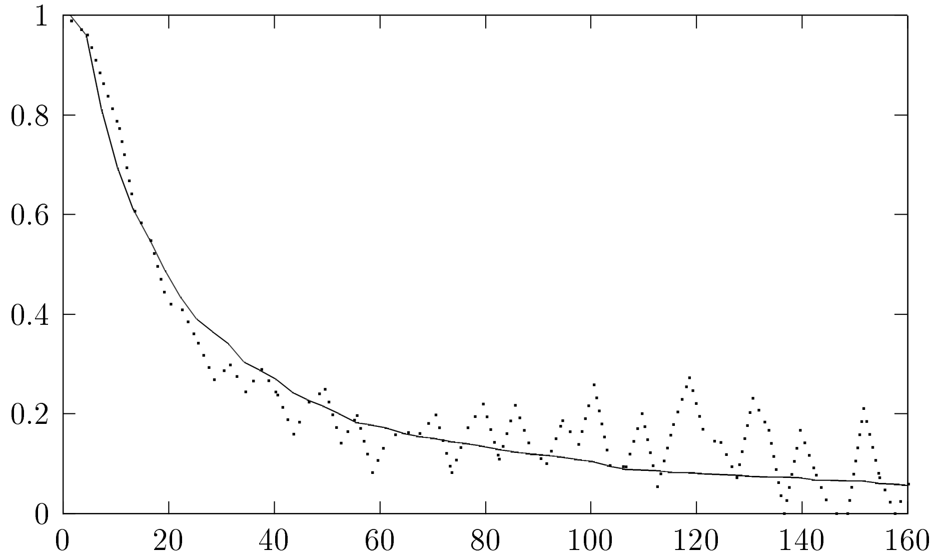

Figure 9: Observed F^{obs} (dotted line) and theoretical $F^{th}_{4.1}$ (solid line) frequencies of payment in cash in function of the price.

payment card. Then a question arises: do TS models fit the payment patterns of people who hold three payment instruments?

In this section, we propose to test the performance of TS models to replicate observed individual and aggregate payment patterns using our data set for three payment instruments. Hence, we limit our sample to individuals who hold three payment instruments: cash, check and payment card.[10]

Let us first test the performance of the TS models at an individual level. For that, we have to identify, as previously, for each individual, the domains of transaction of the three payment instruments. For people who hold three payment instruments such as cash, payment card and check, for the same reasons as for two payment instruments, for each individual, we should observe:

$$P^{max}_{cash} < P^{min}_{payment\,card},\ P^{max}_{payment\,card} < P^{min}_{check} \text{ and } P^{max}_{cash} < P^{min}_{check}, \qquad (3)$$

[10]83% of the people in our sample hold those three payment instruments.

Models	% error (in %)	threshold
CH model (F_l^{th})	3.1	4.1
Naive TS model ($\widehat{F_{18}^{th}}$)	17.1	18
Differentiated TS model ($\widetilde{F_{\bar{l}}^{th}}$)	6.4	$\min \bar{l} = 0.8$ - $\max \bar{l} = 1,488.7$ mean($\bar{l}$)=42.6
Differentiated TS model ($\widetilde{F_{\underline{l}}^{th}}$)	10.8	$\min \underline{l} = 0.7$ - $\max \underline{l} = 634.2$ mean($\underline{l}$) 18.9

Table 1: Summing up of the results.

where P_{cash}^{max} and $P_{payment\,card}^{max}$ are respectively the maximum value of a transaction paid with cash and a payment card and $P_{payment\,card}^{min}$ and P_{check}^{min} are respectively the minimum value of a transaction paid with a payment card and a check.

A first analysis of the payment data shows that some individuals satisfy Equation (3). At the top of Figure 10, we reproduced the payment patterns of such an individual who holds the three payment instruments. We notice three strict domains of transaction: the largest transaction paid cash is strictly lower than the smallest purchase paid with the payment card and the largest transaction paid with cash and the payment card is strictly lower than the lowest transaction paid with check.

However, we also find individuals who do not behave as the TS models predict. For instance, we reproduced the payment pattern of an individual satisfying none of the inequalities of Equation (3) at the bottom of Figure 10. We notice three overlapping domains of transaction. First, we observe that the largest transaction paid cash (around 150 euros) is strictly higher than the smallest purchase paid with the payment card and even higher than the largest purchase paid with the payment card (around 90 euros). Second, we note that the largest transaction paid with the payment card is higher than

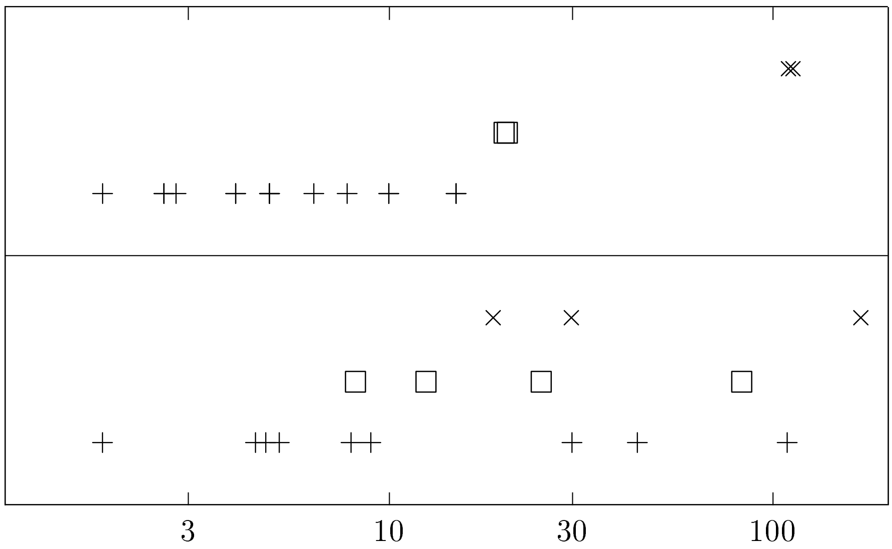

Figure 10: Examples of use of cash (+), payment card (□) and check (×) for two individuals.

the lowest transaction paid with check (around 20 euros). Finally, we remark that the largest transaction paid cash is higher than the smallest transaction paid with check. Hence, this individual does not satisfy Equation (3). More generally, when we extend the analysis to the whole population who hold the three payment instruments and who realize at least one transaction, we find that 48.55% of them do not satisfy Equation (3).

Finally, we could hardly argue that naive TS models replicate the data at an aggregate level with three payment instruments. Indeed, we note in Figure 11 that cash, check and payment card are used for small value transactions as well as for high value transactions. Therefore, our data suggest that there are no strict transaction domains for each payment instrument but rather some overlapping transaction domains.

However, we emphasize two reasons for which our study can hardly be straightforwardly extended for three payment instruments at an aggregate level. The first reasons is valid for all the aggregate models (naive TS, dif-

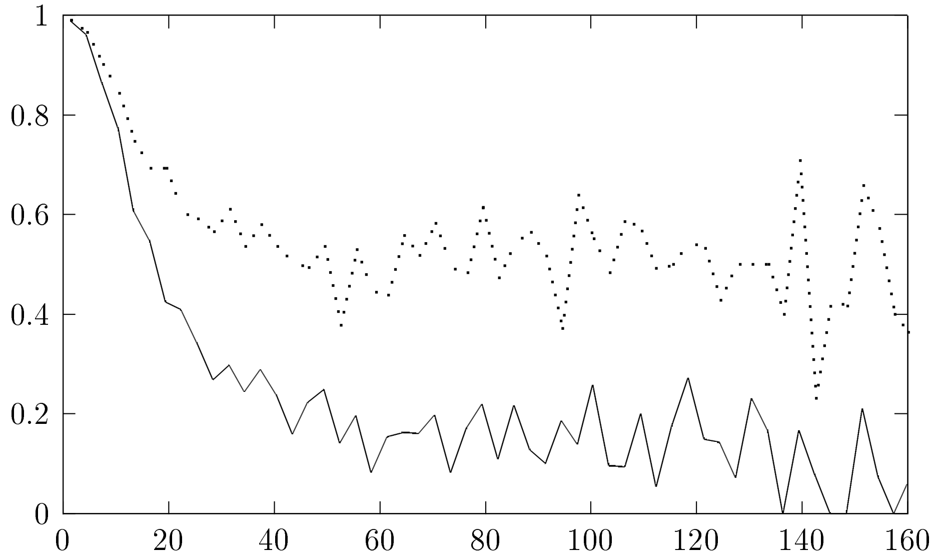

Figure 11: Share of the transactions paid in cash (below solid line), check (between solid and dotted line) and payment card (above dotted line) in function of prices.

ferentiated TS and CH). Our distance to compute how well each theoretical model fits the data can hardly be adapted for more than two payment instruments. We should define a distance that estimates the differences between two pairs of lines. Many possibilities exist and further research would be required in this direction.

The second reason for which extending our results for more than two payment instruments is not straightforward is valid for the differentiated TS and the CH models. Indeed, in order to undertake this generalization, the whole methodology should be significantly different from the ones used in the present study. In the case of the differentiated TS model, the way to find the distribution of the thresholds, that are now two-dimensional, is not obvious. How could we deal with different pairs of thresholds minimizing the number of errors of the TS approach? In the case of the CH model, the way to consider a third payment instrument is not straightforward either.

How can we discriminate between checks and payment cards in a CH model? Those two questions are left for further research.

5 Conclusion

In this article, first, we showed that TS models are not satisfied by the individuals of our data set when they are considered at an individual level. Second, we provided two aggregate TS models and a simple payment rule (CH model). We showed that the latter better fits the observed aggregate payments patterns than the former. Globally, we find that the CH model gives predictions from 2 to 6 times more precise than TS models at an aggregate level and hence may be considered as more efficient than them.

We now emphasize an important feature of the CH model that makes it even more appealing: it is by far less complex to implement than the TS model in its differentiated aggregated form since it relies on a very minimal information basis. Indeed, the only free parameter we need to compute is the threshold l under which the representative agent goes back to withdraw cash. The same feature is shared by the naive TS model where the only free parameter is the threshold L above which all purchases are paid with the alternative payment instruments. As we have shown, the former model performs much better than the latter model at fitting the observed data. On the contrary, the differentiated TS model requires the computation of the approximate threshold for each individual. In our study, since we have 1,386 individuals, we need to have all the information about the payment patterns of each of them. Then, the computation requires a 1,386-elements set as an input. Hence, the differentiated TS model performs less accurately than the CH model and, moreover, shows some obvious informational and computational limits. In the same line, of course, we could imagine some models performing better than the CH model. However, to the best of our understanding, these models would require a much broader informational basis. For instance, a further research would investigate the performance of another model that would mainly consist in the differentiated TS model with a probability distribution among thresholds that would depend on the price faced by the representative agent.

The second limitation of our study concerns the distance we used. As already explained, with this distance, an error of prediction for a given price is weighted by the frequency of the transactions of this price, see Equation (2). Another natural distance would be worth investigating:

$$D(F^{obs}, f^{th}) = \sum_{p \in [0,\infty[} p \frac{n(p)}{N} \mid F^{obs}(p) - f^{th}(p) \mid . \tag{4}$$

With the latter distance, an error of prediction for a given price is weighted by the frequency of the transactions of this price and by the price itself. Hence, the "frequency effect" described in the interpretation of Equation (2) still exists. But there is another effect, the "size effect". According to the latter effect, an error of prediction of 1% for purchases worth 1,000 euros is more important than an error of prediction of 1% for purchases worth 1 euro. Indeed, in the first case, statistically, the error of prediction is 10 euros large (1% of 1,000 euros) whereas in the second case, the error of prediction is 0.01 euro large. If the purpose is to estimate the total amount of cash used in the economy, the distance given in Equation (4) seems more natural. However, the distance shown in Equation (4) overestimates the errors of prediction done for large transactions in comparison with the distance shown in Equation (2). That is why our data set is not large enough to allow us to use the distance shown in Equation (4). For instance, in our data set, we have only one transaction between 20,601 and 20,604 euros. It was paid with an alternative payment instrument, but saying that 0% of the transactions between 20,601 and 20,604 euros is paid in cash is not statistically significative. This is not important if the distance shown in Equation (2) is used since this transaction represents only 1/16,193 of the transactions. But if the distance shown in Equation (4) is used, an error of prediction for this very transaction is much more important than an error for any other price. Hence, the result would be greatly influenced by a data that is not statistically significant. Then, in order to use the distance shown in Equation (4), a larger data set would be necessary.

A Descriptive statistics on transactions

	Cash	Alternative Payment Instruments	Total
Nb. of transactions	10,419 (64.3%)	5,774 (35.7%)	16,193 (100.0%)
Average value	10.8	68.0	31.2
s.d.	27.2	363.1	219.7
Max Nb. of transactions for one indiv.	34	21	39
Nb. of indiv. making at least 1 transaction	1,309	1,165	1,386

Table 2: Descriptive statistics on transactions.

B Descriptive statistics on withdrawals

	Cash
Nb. of withdrawals	1,785
Average value	71.4
s.d.	105.6
Min - Max	10 -2,500

Table 3: Descriptive statistics on withdrawals.

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