Interchange fees and incentives to invest in the quality of a payment card system*

Marianne Verdier†

October 22, 2007

Abstract

In this paper, I analyse the optimal interchange fee in a payment card system where two monopoly banks (an Issuer and an Acquirer) can invest to deliver a better quality of service to their customers. If the level of quality is exogenous, I extend Baxter (1983)'s model, by showing that the optimal level of interchange fee, which is equal to the Acquirer's margin (the merchant's bank), depends on the level of quality delivered by the payment system. If the level of quality is endogenous, the level of the interchange fee which maximises banks' joint profits depends on the relative contributions of banks to quality investments, and the relative perceptions of quality improvements on each side of the market. I show that, in some cases, because of the strategic effects, the payment platform may choose an interchange fee which is strictly lower than the Acquirer's margin in order to stimulate the Acquirer's investments.

JEL Codes: G21, L31, L42.

Keywords: Payment card systems, interchange fees, two-sided markets, investments in quality.

*I would like to thank Marc Bourreau for helpful comments and discussions, and audiences at the CREST-LEI seminar, the PSE-TOM seminar, and at the 2007 EARIE Conference.

†Author's affiliation: ENST, Department of Economics and Social Sciences, 46 rue Barrault, 75634 Paris Cedex 13, France; E-mail: marianne.verdier@enst.fr.

1 Introduction

Payment cards have become very popular over the last twenty years. For instance, in Europe, a total of 23 billion card payments are made annually, with an overall value of 1,350 billion Euros.[1] In several countries, the usage of payment cards has surpassed the usage of cash for retail payments. For instance, in the United Kingdom, total spending on payment cards outstripped cash spending for the first time in 2004, and the average adult owns 3.6 payment cards.

The success of payment cards can be related to their quality and convenience. In many countries, payment card systems have strived to improve the level of quality of payment card services, both for consumers and merchants, such as the delay and quality of transaction processing, the quality of payment terminals and communication facilities, the quality of security measures, the information about acceptation points for consumers, and the information about fraudsters for merchants. The level of quality of payment cards services depends on the investments made by the banks or imposed by the payment system.

This paper tries to determine the optimal price structure of a payment system when banks can invest in quality. In open-loop payment systems, such as Visa and MasterCard, a key element which influences the price structure and the volume of transactions made by cards is the payment of an interchange fee by the merchant's bank (the Acquirer) to the cardholder's bank (the Issuer). My purpose is to study the effect of the interchange fee on banks' incentives to invest in the quality of payment card services.

To that end, I propose a framework to study quality investments in a payment system, and their effects on the optimal interchange fee. My aim is to determine if a payment system, which has the opportunity of improving its quality of service, should adjust its interchange fee. I show that it depends on the asymmetries between each side of the market, namely the relative contribution of each bank to investments in quality, and the relative effect of quality improvements on consumers and merchants.

Recent discussions between the European Commission and the banks have shown that the relationship between the level of the interchange fee and the quality of the payment system services is a relevant policy issue, especially in Europe. On the one hand, in the Interim Report on Payment Cards released in April 2006, the European Commission called into question the role of interchange fees, which could lead to excessive profits in the issuing industry. On the other hand, several banks argued in their responses to the Interim Report that the profitability

[1] Source: Interim Report on Payment Cards, European Commission (April 2006).

measured was exaggerated "by ignoring the cost of financing investments in a payment card business".[2] According to the banks, interchange fees are necessary to provide them with the right incentives to share investments, which improves the quality of the services provided by the payment system. Since additional investments will be needed to ensure interoperability between payment card schemes for the creation of the Single Euro Payments Area (SEPA),[3] a reflexion about the relationship between the level of the interchange fee and the investments in quality should provide interesting insights to the debate between the banks and the competition authorities.

In this paper, I model a payment card system as a two-sided platform, which organizes the interactions between a monopolistic Issuer and a monopolistic Acquirer by choosing an interchange fee, paid by the Acquirer to the Issuer. In my setting, the Issuer provides payment card services to heterogeneous cardholders, while the Acquirer provides payment card acceptance facilities and services to homogeneous merchants. If banks have the possibility to invest in quality, I assume that the quality of the payment card service is defined as a weighted sum of contributions from the Issuer and the Acquirer.

The observation of payment card systems shows that both banks contribute to a better quality of service for consumers and merchants. For instance, the improvement of the security requires investments from the Issuer and the Acquirer. The Issuer invests to improve the quality of the chip, to gather data about card numbers, cardholders, fraudulently used cards, while the Acquirer invests to install specific software on merchants electronic payment terminals, which enables them to obtain information from the authorisation network. To improve the response of the authorisation network, the Issuer and the Acquirer must install several network lines, increase the size of their database, improve the quality of their electronic equipment. The guarantee of payment for merchants requires a screening of cardholders which can only be completed by the Issuer. These latter examples show that quality investments do not have necessarily the same impact on consumers and merchants. Therefore, I model the impact of quality investments on each side of the market with two different parameters. I assume that, if

[2] This was the response of Barclays, which carries on by saying that "the continued development of EMV Chip+PIN and secure e-commerce technology which are both important ingredients in SEPA are dependent on the continuation of current, successful payment card business models". Citybank added that interchange fees provide "incentives for innovation and investments". La Caixa pointed out that investments to implement the EMV technology amounted to 40,295,500 Euros until 2010 and that 25,000,000 Euros had already been invested in updating ATM and terminals. Swedbank argued that interchange fee are needed to ensure "investments in build-up, maintenance and continuing development of payment card services". See: "http//ec.europa/comm/competition/antitrust/others/sector_inquiries/financial_services/rep_report_1.html".

[3] The aim of the Single Euro Payments Area si to build a zone in which consumers will be able to make and receive payments in Euro under the same conditions, obligations and rights, regardless of their location.

consumers benefit from a higher level of quality, the demand for payment card services increases on the consumer side. Since merchants are homogeneous, I assume that, if merchants benefit from a higher level of quality, their willingness to pay for payment card acceptance facilities increases.

Many banks cannot coordinate on the choice of an optimal level of quality of service, because they are sometimes specialised either on the issuing or the acquiring services. Some payment platforms correct this type of externality by imposing on banks a minimum level of investments in quality. However, the payment platform cannot always sign contracts with banks which will ensure that they will make the appropriate level of effort in order to reach the level of quality needed to optimize the profits of the system.

Therefore, my model focuses on the choice of the levels of quality once the interchange fee has been fixed by the payment platform. I begin the analysis with a benchmark case, in which the level of quality is exogenous. I show that, in this case, it is optimal for the payment system to choose an interchange fee which is equal to the Acquirer's margin per transaction, like in Baxter (1983)'s model. But, unlike Baxter's interchange fee, in my model, the optimal level of interchange fee takes into account the level of quality of service.

If the levels of quality are endogenous, I show that the profit maximising interchange fee may be strictly lower than the Acquirer's margin, if investments in quality impact relatively more the consumer side. This is because, in some cases, the Acquirer's investments in quality and the interchange fee become strategic substitutes. Therefore, sometimes, the payment platform will make more profit by decreasing the level of interchange fee to provide the Acquirer with incentives to invest in quality, which impact the cardholders' demand positively. If investments impact relatively more the merchant side, the optimal interchange fee is the maximum interchange fee that satisfies the budget constraint of the Acquirer.

Afterwards, I take the point of view of a social planner, who tries to determine the welfare maximising interchange fee. I show that, if investments in quality impact relatively more the merchant side, the welfare maximising and the profit maximising interchange fee are equal. On the contrary, if investments impact relatively more the consumer side, I derive the conditions under which there is an overprovision of payment card services when the interchange fee is chosen by a profit maximising payment platform.

Then, I analyse the case in which the payment platform can also choose the levels of quality. The payment platform solves the coordination problem faced by the banks when they make their investment decisions. I show that, in this case, the interchange fee is fixed at the maximum level compatible with the budget constraint of the Acquirer, which is not always the case if banks cannot cooperate on investments, because of the coordination problem. However, the level of the interchange fee with cooperation on investments may be higher or lower than the profit maximising interchange fee if banks cannot cooperate on the quality choices.

Finally, I discuss the main assumptions of my model: the market structure and the homogeneity of merchants. I also discuss the optimal structure of the interchange fee. When the Acquirer's budget constraint is binding, the payment platform can increase its profit by choosing a two-part tariff structure for the interchange fee, which is not the case in Baxter (1983)'s model. My model gives also the intuition that, if the market is more concentrated on the acquisition side, the optimal interchange fee may be decreased to obtain a higher level of quality. I also extend Schmalensee (2002)'s model by showing that, if merchants are heterogeneous, if banks are monopolists and if the level of quality is exogenous, the interchange fee should be chosen so as to equalize banks' marginal costs, net of the marginal benefits of investments in quality. Unfortunately, our results cannot be generalised easily if the levels of quality are endogenous.

The existing literature on payment systems does not take into account the possibility for banks to invest in the quality of the payment system. Many authors (see among others: Baxter 1983, Rochet and Tirole 2002, Schmalensee 2002, Wright 2004) have analysed the role of interchange fees, which is to balance demands from the two sides of the market, and to optimise the functionning of the payment system. They have also compared the socially optimal interchange fee with the fee chosen by the payment system to maximise its profits, in order to assess the welfare effects of this "cooperation between competitors".[4] These models involve various assumptions about consumers, merchants, and competition between banks.[5] However, none of them takes into account the additional utility generated by a better level of quality, as it is done in my paper.

The closest paper to mine is the paper by Rochet and Tirole (2007) in which the level of quality perceived by the cardholder is internalised by the merchant for strategic purposes. In their model, merchants accept cards if the merchant commission is lower than their card acceptance benefits plus a parameter that models the consumers' awareness of the quality of service. The perspective of my paper is different, because I model a specific perception of the quality of service on each side of the market, and link it to banks' investments, which is not done in Rochet and Tirole's paper. In my paper also, merchants are not strategic, so they do not internalise the level of quality perceived by consumers. This assumption enables me

[4] Rochet and Tirole (2002).

[5] In the literature, consumers are assumed to be either homogenous or heterogeneous. Strategic merchants can sometimes use cards to differentiate themselves from their competitors (Rochet and Tirole, 2002). Different kinds of competition between issuers on the one hand, and acquirers on the other hand are modelled. For a review of the literature, see Rochet (2003).

to understand better the influence of the strategic interactions between banks when the latter choose how much to invest in quality.

The rest of the paper is organised as follows. In section two, I start by presenting the model and the assumptions. In section three, I solve for the equilibrium of the game. In section four, I study an example with quadratic cost functions. In section five, I give extensions of the results obtained in section four with other market structures, and heterogeneous merchants. I also discuss the optimal structure of the interchange fee. Finally, I conclude.

2 The model

I model an open-loop payment system as a two-sided market. On the issuing side, a monopolistic Issuer provides payment card services to heterogeneous cardholders. On the acquiring side, a monopolistic Acquirer provides payment card terminals and payment acquisition services to homogeneous merchants. The payment platform organises the interactions between the Issuer and the Acquirer by setting an interchange fee which is paid by the Acquirer to the Issuer on a per-transaction basis.

The respective benefits of card usage for consumers and card acceptance for merchants depend on the quality of the services offered by the payment system. The overall quality of the payment system depends on banks' investments in quality.

Consumers: Each consumer owns a payment card and another payment instrument.[6] A consumer is characterised by his benefit, b_B, of using a payment card rather than the other payment instrument. The benefit b_B is assumed to be uniformely distributed over $[0,1]$, which implies that consumers differ across their card usage benefits. One interpretation is that they may attach different values to the convenience of using a payment card rather than cash.

I assume that, if the payment system delivers a higher quality of service, the consumers' benefits of card usage increase. For instance, if investments are made to decrease the duration of a card transaction at the point of sales, consumers may find it more convenient to use their cards rather than cash. Denoting the quality of the payment system by θ, the card usage benefit of a consumer of type b_B becomes $b_B + \alpha_B\theta$, where α_B is a parameter that represents the constant marginal positive effect of quality on usage benefits. Under this assumption, consumers with different types benefit equally from a better quality of service.

[6] In the model, I consider cardholding decisions as exogenous, and focus on the choice of using a payment card rather than another payment instrument, like cash, for instance, at the point of sales.

Merchants: I suppose that merchants are homogeneous as regards to their card usage benefit, which is denoted by b_S (with $b_S \geq 0$). Merchants are not strategic. I also assume that the quality of the payment system increases their benefits of card usage. If the quality of the payment system is θ, merchants' benefit of card usage becomes $b_S + \alpha_S\theta$, where α_S is a parameter that represents the marginal positive effect of quality on merchants'utility.

Notice that the marginal effects of quality on the benefits of consumers and merchants differ if $\alpha_S \neq \alpha_B$.

Banks: The Issuer (I) and the Acquirer (A) are monopolists. For each transaction, the Issuer charges card-users with a fee, f, and the Acquirer charges merchants with the commission, m. The Acquirer pays to the Issuer a per-transaction interchange fee, denoted by a. The interchange fee can be either positive or negative. Banks' have constant marginal costs c_i per transaction, for $i = I, A$, and the total marginal cost of the system is defined as $c = c_A + c_I$. The per-transaction margins are denoted by M_i, for $i = I, A$. Profits are denoted by π_I and π_A.

Banks can invest to improve the quality of payment card services. The level of quality set by bank i is denoted by θ_i, for $i = I, A$. The cost of a quality level of θ_i is denoted by the function $C_i(\theta_i)$ where $i = I, A$. I assume that $C_i(\theta_i)$ is twice differentiable and that $C_i^{'}(\theta_I) > 0$ and $\mathrm{C}_i^{''}(\theta_I) > 0$. The quality of payment card services is modelled as a combination of the quality produced by the Issuer, θ_I, and the quality produced by the Acquirer, θ_A:

$$\theta = \lambda_I\theta_I + \lambda_A\theta_A,$$

where λ_I and λ_A reflect the respective contribution of the Issuer and the Acquirer to the quality of the payment system ($\lambda_i \geq 0$ for $i = I, A$). I denote the total cost of the payment system by $C_S(\theta) = C_I(\theta_I) + C_A(\theta_A)$.

If all merchants accept cards, the Issuer and the Acquirer make profits

$$\pi_I = VM_I(a, f) - C_I(\theta_I),$$

and

$$\pi_A = VM_A(a, m) - C_A(\theta_A),$$

where V represents the transaction volume.

If no merchant accepts cards, banks make no profits, i.e., $\pi_i = 0$ for $i = I, A$.

Payment system: The payment system (S) chooses the interchange fee, a, which maximises the sum of banks' profits, $\pi_S = \pi_I + \pi_A$. I assume that surcharges are not allowed, which means that merchants are forbidden to charge consumers a higher retail price if the latter use their payment cards. The total margin of the payment system is denoted by $M_S = M_I + M_A$.

Finally, I define the social welfare, W, as the sum of consumers' surplus, S_C, merchants' surplus, S_M, and banks' profits, $\pi_I + \pi_A$.

I also make the following assumptions:

A1 $b_S \geq c_A$.

A2 At the equilibrium, some consumers use their cards but not all.

A3 $C_I''(\theta_I) > \dfrac{(\alpha_B \lambda_I)^2}{2}$ and $C_A''(\theta_A) > (\alpha_S \lambda_A)^2$ for all $\theta_I, \theta_A \geq 0$.

The first assumption means that merchants accept cards if the Acquirer prices the transaction at its marginal cost. The second assumption states that the market is not covered at the equilibrium, which enables me to analyse the relationship between the interchange fee and the expansion of the payment card market. The third assumption ensures that the second-order conditions are satisfied in the maximisation problems.

The timing of the game is as follows:

1. The payment platform chooses the interchange fee, a, which maximises the joint profits of the banks.
2. Banks decide simultaneously and non-cooperatively on the levels of quality, θ_I and θ_A.
3. Banks choose simultaneously and non-cooperatively their transaction fees, f and m.
4. Consumers decide whether or not to use their payment cards. Merchants decide whether or not to accept cards.

I focus on the choice of the levels of quality once the interchange fee has been fixed by the payment platform. In practice, the level of the interchange fee is not reajusted very frequently,[7] which justifies the choice of this timing for the game. I study how the choice of the interchange fee impacts the levels of investment that are decided by banks, non cooperatively, without any intervention of the payment platform.

I look for the subgame perfect equilibrium, and solve the game by backward induction.

[7] For example, Visa has not changed the level of its interchange fee in Europe between 2002 and 2007.

3 The equilibrium

3.1 Stage 4: card usage and card acceptance

For given θ and m, a merchant accepts cards if

$$b_S + \alpha_S \theta \geq m. \tag{1}$$

Since all merchants have the same benefit b_S, all merchants accept cards if (1) holds, and no merchant accepts cards otherwise.

A consumer of type b_B wants to use his card if

$$b_B + \alpha_B \theta \geq f. \tag{2}$$

If all merchants accept cards, the transaction volume is equal to the percentage of consumers willing to use their cards, such that[8]

$$V = V(\theta, f) = P(b_B + \alpha_B \theta \geq f).$$

Notice that, since b_B is uniformely distributed over $[0;1]$, if $f - \alpha_B \theta \in (0;1]$, the market is not covered, and

$$V(\theta, f) = 1 + \alpha_B \theta - f. \tag{3}$$

At the equilibrium of stage 4, there are two possible cases. Either (1) holds such that all merchants accept cards, and $V(\theta, f)$ consumers use their cards, or no merchant accepts cards and all consumers use cash.

3.2 Stage 3: transaction fees.

Each bank chooses the transaction fees that maximise its profit. There are two cases: either m is set such that all merchants accept cards, or m is too high, such that no merchant accept cards. Let me start by the first case. If all merchants accept cards, banks' profits are expressed as follows:

$$\pi_I = V(\theta, f) M_I(a, f) - C_I(\theta_I),$$

and

[8] The market size is normalised to 1 in the model.

$$\pi_A = V(\theta, f)M_A(a, m) - C_A(\theta_A),$$

where $V(a, \theta, f)$ is given by (3). All merchants accept cards if the fee m is not too high. Since the Acquirer's profit, increases with m, for given a, θ_I, and θ_A, the Acquirer sets the maximum fee such that (1) holds, that is,

$$m(\theta) = b_S + \alpha_S\theta. \tag{4}$$

Given that cards are accepted by merchants, the Issuer chooses the consumer fee, f, that maximises its profit. Solving the first-order condition

$$\frac{\partial \pi_I}{\partial f} = 1 + \alpha_B\theta - a + c_I - 2f = 0,$$

the optimal fee is:[9]

$$f^*(a, \theta) = \frac{1 + \alpha_B\theta - a + c_I}{2}. \tag{5}$$

For the Issuer, the choice of the optimal customer fee involves a trade-off between a smaller margin per transaction and a higher transaction volume. Replacing for $f^*(a, \theta)$ in (3) yields the transaction volume at the equilibrium of the subgame, that is,

$$V(\theta, f^*(a, \theta)) = \frac{1 + \alpha_B\theta + a - c_I}{2} = V(a, \theta).^{10} \tag{6}$$

Notice that the transaction volume is increasing with the quality of the payment sytem, θ, and the interchange fee, a. Banks' margin at the equilibrium of stage 3 are expressed as follows:

$$M_I(a, \theta) = \frac{1 + \alpha_B\theta + a - c_I}{2} = V(a, \theta), \tag{7}$$

and

$$M_A(a, \theta) = b_S + \alpha_S\theta - a - c_A. \tag{8}$$

If cards are not accepted by merchants, no consumer uses his card, and banks make no profits. This does not constitute an equilibrium, as the Acquirer could raise its profit by choosing a merchant fee that satisfies (1).

[9] The second-order condition is satisfied.

[10] In the following sections, to simplify the exposition, I will abuse of the notation V, and write $V(\theta, f^*(a, \theta)) = V(a, \theta)$.

Notice that banks exert price externalities on each other. For instance, if the Acquirer chooses m such that no merchant accepts cards, the Issuer makes no profits, and loses all its consumers. Likewise, if the Issuer chooses a higher consumer fee, the transaction volume becomes lower, which reduces the profit of the Acquirer.

3.3 Stage 2: levels of quality

At stage 2, banks decide simultaneously and non cooperatively on their levels of quality. I start by looking at the properties of the best response functions, and then, I determine the equilibrium of the subgame. Bank i chooses the level of quality θ_i that maximise its profit π_i, given θ_j and a, for $(i, j) \in \{A, I\}^2$ and $i \neq j$. Its best response function is denoted by $R_i(a, \theta_j)$.

Lemma 1 *Assume that θ_I and θ_A constitute an equilibrium. Then, we have*

$$C_I'(\theta_I) = \alpha_B \lambda_I V(a, \theta), \tag{9}$$

and

$$C_A'(\theta_A) = \alpha_S \lambda_A V(a, \theta) + \frac{\alpha_B \lambda_A}{2}(b_S + \alpha_S \theta - a - c_A). \tag{10}$$

Proof. See Appendix A. ■

Each bank chooses its best response such that the marginal costs of investments are equal to the marginal benefits of a higher level of quality θ. The marginal benefits of a higher quality can be divided into two parts (see the table below): the marginal benefits obtained through higher prices, and the marginal benefits obtained because of an increase of the transaction volume.

First, investments in quality increase banks' margins per transaction, if their clients benefit from a higher quality of service.[11] Since banks have market power, they can raise their prices following an increase in the quality of service. For instance, we already noted from (5) that the customer fee increases with θ. We also proved in (4) that the Acquirer extracts all the surplus of the merchants, by charging them with a price, m, that reflects exactly their benefits of card acceptance. We summarise the marginal benefits obtained through higher prices in the following table:

[11] From (7), we know that the Issuer's margin per transaction increases with θ if $\alpha_B \neq 0$, and from (8), that the Acquirer's margin per transaction increases with θ if $\alpha_S \neq 0$.

Table 1: Marginal benefits of investments in quality

	Issuer	Acquirer
Marginal benefits obtained through higher prices.	$\frac{\alpha_B \lambda_I}{2} V(a, \theta)$	$\alpha_S \lambda_A V(a, \theta)$
Marginal benefits obtained through higher volumes.	$\frac{\alpha_B \lambda_I}{2} V(a, \theta)$	$\frac{\alpha_B \lambda_A}{2}(b_S + \alpha_S \theta - a - c_A)$

Second, from (6), we know that the transaction volume increases when banks invest more in quality, because the Issuer does not extract all the surplus from customers. We already noticed that, since merchants are homogeneous, the transaction volume depends only on cardholders' demand. This is an important source of asymmetry in our model. The Issuer chooses its level of quality according to the benefits generated on his side of the market, namely the cardholder side, while the Acquirer takes into account both sides of the market, which is reflected by the presence of α_B and α_S in (10).

In the setting, banks exert externalities on each other, by choosing their contribution to the overall level of quality, θ, which can be viewed as a public good.

It is important to note that, in the model, externalities are asymmetric. The first reason is that consumers are heterogeneous, while merchants are homogeneous, so, orginally, both sides of the market are not symmetric.[12] The monopolist Acquirer is able to charge merchants with the exact amount of quality increase, $\alpha_S \theta$, while the Issuer leaves some surplus to cardholders, because it controls the level of the transaction volume.

The second reason is that banks' contributions to the level of quality of the payment system are different if $\lambda_I \neq \lambda_A$. For instance, if $\lambda_i = 0$ and $\lambda_j > 0$, bank i may enjoy as a free-rider the benefits of bank j's investments. Therefore, in this case, bank j exerts a positive externality on bank i.

The third reason is that quality levels may be perceived differently by consumers and merchants if $\alpha_B \neq \alpha_S$. For instance, if $\alpha_B = 0$, cardholders do not benefit from a better quality of service, and the Issuer does not invest in quality. However, if $\alpha_S > 0$, the Acquirer and the merchants would benefit from a higher level of quality, so the Issuer exerts a negative externality on the other side of the market.

Lemma 2 *The levels of qualities, θ_I and θ_A, are strategic complements.*

Proof. See Appendix B. ■

If the Acquirer chooses a higher level of quality, this increases the transaction volume, and the Issuer's marginal benefits of quality investments. From (9), we know that the Issuer's level

[12] In section 5.2, I shall extend the model by introducing heterogeneity among merchants.

of quality is chosen such that its marginal benefits are equal to its marginal cost of quality. In our model, bank's marginal costs of quality are increasing with the level of quality. So, if the Acquirer invests more, the Issuer responds by choosing a higher level of quality. The intuition is exactly the same for the Acquirer: when the Issuer chooses a higher level of quality, this raises the Acquirer's marginal benefits of investments, so the Acquirer also chooses a higher level of quality at the equilibrium.

Lemma 3 *The Issuer's quality, θ_I, and the interchange fee, a, are strategic complements. If $\alpha_B \leq \alpha_S$, the Acquirer's quality, θ_A, and the interchange fee, a, are strategic complements, otherwise, they are strategic substitutes.*

Proof. See Appendix C. ■

The intuition behind this result is the following. We already saw that the Acquirer's incentives to choose a high level of quality depend on two effects:

- the marginal benefits obtained when quality increases, $\alpha_S \lambda_A$, multiplied by the transaction volume (gains from the merchant side),
- and the marginal volume of transactions generated by a higher quality, $(\alpha_B \lambda_A)/2$, multiplied by the margin M_A (gains from the consumer side).

If the interchange fee increases slightly from a to $a + da$, all other things being equal, the transaction volume rises by $da/2$, while the margin M_A diminishes by $-da$. That is, the Acquirer makes marginally positive profits from the merchant side, while it loses marginally from the consumer side. If $\alpha_B \leq \alpha_S$, merchants benefit marginally more from a higher quality than consumers, which implies that the Acquirer's marginal profits from the merchant side compensate its marginal losses from the consumer side. So, if $\alpha_B \leq \alpha_S$, then θ_A and a are strategic complements, and otherwise, they are strategic substitutes.

The Issuer's incentives to choose a high quality depend only on its gain from the consumer side. We saw in the previous subsection that the higher the transaction volume and the Issuer's margin are, the higher is the level of quality θ_I. A slight increase in the interchange fee generates at the same time a higher transaction volume and a higher margin for the Issuer. Therefore, θ_I and a are unambiguously strategic complements.

Equilibrium of the investment subgame: The levels of quality at the equilibrium are denoted by $\theta_I^*(a)$, $\theta_A^*(a)$, and $\theta^*(a)$, where:

$$\theta_A^*(a) = R_A(a, \theta_I^*(a)), \tag{11}$$

and

$$\theta_I^*(a) = R_I(a, \theta_A^*(a)). \tag{12}$$

Lemma 4 *If $\alpha_B \leq \alpha_S$, banks' levels of quality increase with the interchange fee, that is $(\theta_A^*)'(a) \geq 0$ and $(\theta_I^*)'(a) \geq 0$.*
Otherwise, the sign of $(\theta_A^*)'(a)$ and $(\theta_I^*)'(a)$ can be either positive or negative.

Proof. See Appendix D. ■

The result is quite intuitive if $\alpha_B \leq \alpha_S$. In that case, we know that the qualities and the interchange fee are strategic complements. At the same time, qualities are always strategic complements. So, if the interchange fee increases slightly, the Issuer's incentives to invest in quality increase, while the Acquirer's chooses a higher level of quality because of the effect of strategic complementarity. The same reasoning can be applied to the Issuer's choice of θ_I.

If $\alpha_B > \alpha_S$, the level of quality chosen by the Acquirer, θ_A , and the interchange fee, a, are strategic substitutes, while θ_I and a are strategic complements. If the interchange fee increases slightly, the Acquirer tends to reduce its level of quality, because the marginal increase of the transaction volume and the marginal benefits obtained on the merchant side are not sufficient to compensate the reduction of its margin. However, the levels of quality are strategic complements, and the Issuer chooses a higher level of quality. This tends to increase the level of quality chosen by the Acquirer. Depending on how both effects compensate each other at the equilibrium, the levels of quality chosen by the Issuer and the Acquirer can either increase or decrease. In section 4, I will provide an example that illustrates this case.

3.4 Stage 1: choice of the optimal interchange fee

Should a payment system choose a higher interchange fee when banks can invest in quality? In this section, I try to compare the profit maximising interchange fee with a benchmark case, in which the level of quality is exogenous. In another benchmark, I determine the optimal levels of quality if they are chosen by the payment platform. Afterwards, I take the point of view of a social planner, who tries to determine at stage one if the interchange fee choosen by a profit maximising payment platform leads to an overprovision of payment card services.

3.4.1 A benchmark: optimal interchange fee if the level of quality is exogenous

In this section, I assume that banks do not have the possibility to determine the level of quality of the payment system, that is, the parameter θ is exogenous. The payment system chooses the optimal interchange fee a^E so as to maximise banks' joint profits,[13] $\pi_S^E(a) = \pi_I^E(a) + \pi_A^E(a)$.

The profit of bank i is $\pi_i^E(a) = V^E(a)M_i^E(a)$ for $i \in \{A, I\}$, where

$$V^E(a) = M_I^E(a) = \frac{1 + \alpha_B\theta + a - c_I}{2},$$

and

$$M_A^E(a) = b_S + \alpha_S\theta - c_A - a.$$

Proposition 1 *If the level of quality θ is exogenous, the optimal interchange fee $a^E(\theta)$ paid by a monopolist Acquirer to a monopolist Issuer is equal to the Acquirer's margin, that is,*

$$a^E(\theta) = b_S - c_A + \alpha_S\theta.$$

The Acquirer makes no profit, whereas the Issuer makes strictly positive profits.

The optimal interchange fee $a^E(\theta)$ maximises the social welfare W under the budget constraints.

Proof. The reader can refer to Appendix E. ■

Since the Acquirer always chooses his fee such that all merchants accept cards, the only way for the payment system to increase the transaction volume is to stimulate consumers' demand. In this setting, because of merchants' homogeneity, the interchange fee does not balance demands between each side of the market, as in the case in other models from the literature (Schmalensee 2002, Wright 2004). That is why the payment system chooses a positive interchange fee, paid by the Acquirer to the Issuer, which is equal to the per-transaction margin of the Acquirer. It is optimal for the payment system to transfer the Acquirer's margin to the Issuer, because the latter can stimulate cardholder's demand by lowering transaction fees.

Another way of understanding this result is to note that the Issuer has a monopoly of access to cardholders' demand. Because of this competitive bottleneck, it is optimal for the payment system to select the maximum interchange fee compatible with non negative profits for the Acquirer. Notice that this is also the case in Baxter's model (1983), even if there is perfect competition between banks. In my model, Baxter's interchange fee, $b_S - c_A$, is obtained if $\theta = 0$. My analysis extends Baxter's model by showing that the level of quality of the payment system influences the choice of the optimal interchange fee. Notice that, in this case, the Issuer takes all the marginal benefits of investments.

In the following section, I will take as a benchmark the interchange fee $a^E(\theta)$, which is equal to the Acquirer's margin. I will try to determine if the optimal interchange fee can become

[13] The letter "E" stands for exogenous.

lower than the Acquirer's margin if banks have the possibility to invest in quality.

3.4.2 Optimal interchange fee with banks' investments

In this section, I consider the possibility of quality investments. I start by studying the effect of the interchange fee on banks' profits. Then, I analyse the choice of the profit maximising interchange fee by the payment system, and compare it to the benchmark interchange fee obtained if the quality is exogeneous. Finally, I compare the profit maximising interchange fee, and the welfare maximising interchange fee.

Impact of the interchange fee on banks' profits: The payment system chooses the interchange fee, denoted by a^P, which maximises banks' joint profits:

$$\pi_S(a, \theta_I^*(a), \theta_A^*(a)) = \pi_I(a, \theta_I^*(a), \theta_A^*(a)) + \pi_A(a, \theta_I^*(a), \theta_A^*(a)), \tag{13}$$

under the constraint that the Acquirer's profit must remain positive ($\pi_A \geq 0$).

I assume that the second-order conditions of profit maximisation are verified. The first-order condition is

$$\frac{d\pi_S}{da} = \frac{\partial \pi_I}{\partial a} + \frac{\partial \pi_A}{\partial a} + \frac{\partial \pi_I}{\partial \theta_A}\frac{\partial \theta_A^*}{\partial a} + \frac{\partial \pi_A}{\partial \theta_I}\frac{\partial \theta_I^*}{\partial a} = 0.$$

The interchange fee has both a direct effect and a strategic effect on banks' profits. In appendix F, I determine the direct and the strategic effects which are summarised in the following table:[14]

Table 2: The strategic effects.

Bank	Direct effect	Strategic effect
Issuer	Positive	Sign of $(\theta_A^*)'(a)$
Acquirer	Positive or negative	Sign of $(\theta_I^*)'(a)$
Joint profits	Positive	Positive (resp., negative) if $(\theta_i^*)'(a) \geq 0$ (resp., ≤ 0).

The direct effect measures how banks' profits react to a small increase of the interchange fee, if the levels of quality are exogenous. The direct effect for the Issuer is always positive, because the interchange fee increases both its margin, M_I, and the transaction volume. For the Acquirer, the direct effect may be positive or negative, because the interchange fee has a positive impact on the transaction volume, while it has a negative impact on its margin, M_A.

The strategic effect measures how bank i's profit reacts to an increase in the interchange fee through its strategic interaction with bank j at stage 2, when the levels of quality are chosen.

[14] In the following table, I assume that $a \leq b_S + \alpha_S\theta - c_A$.

The sign of the strategic effect for bank i is the same as the sign of $(\theta_j^*)'(a)$, for $(i,j) \in \{A, I\}^2$ and $i \neq j$.

If $\alpha_B = 0$, there are no strategic effects, because the level of quality has no impact on consumers' demand.

If $\alpha_B \in (0; \alpha_S]$, the strategic effects are positive, because banks' investments in quality increase with the interchange fee. Since the effect of quality investments is relatively stronger on the merchant side, a higher interchange fee provides the Acquirer with incentives to choose a higher level of quality. This is because the Acquirer can recover the costs of investment by charging higher prices, while obtaining higher transaction volumes thanks to the effect of the interchange fee on consumers' demand. Since the levels of quality are strategic complements, the Issuer selects also a higher level of quality. By choosing a higher level of quality, the Issuer increases the transaction volume and the Acquirer's margin. The profit of the Acquirer increases with the investments of the Issuer. The same reasoning is true for the profit of the Issuer.

If $\alpha_B > \alpha_S$, the strategic effects may be either positive or negative. In this case, the effect of quality investment is relatively stronger on the consumer side. If the interchange fee increases slightly, the marginal increase of the transaction volume and the marginal benefits obtained on the merchant side are not sufficient to compensate the marginal reduction of the Acquirer's margin. As we noted before, both banks can either react by chosing a higher level of quality or a lower level of quality, depending on how the strategic effects compensate each other. If bank i reduces its level of quality, this has a negative impact on the profit of bank j, because of a lower transaction volume, and a lower margin.

Choice of the optimal interchange fee: I can now analyse the optimal interchange fee.

Proposition 2 *If $\alpha_B \leq \alpha_S$, the Acquirer makes no profit, $\pi_A = 0$. The profit maximising interchange fee is the highest a^P that satisfies*

$$\pi_A(a^P, \theta_I^*(a^P), \theta_A^*(a^P)) \geq 0. \tag{14}$$

If $\alpha_B > \alpha_S$, we have $\pi_A \geq 0$. If $\pi_A > 0$, the profit maximising interchange fee, a^P, is strictly lower than the Acquirer's margin, that is,

$$a^P < a^E(\theta^*(a^P)) \tag{15}$$

Proof. See Appendix G. ■

Notice that we do not solve the case in which $(\theta_i^*)'(a) \geq 0$ and $(\theta_j^*)'(a) \leq 0$, for $i \neq j$ and $(i,j) \in \{A, I\}^2$. In that case, it is not possible to determine if the constraint is binding, and to compare a^E and a^P. It depends on how the strategic effects compensate each other. In section 4, I will provide more structure to the model to investigate this case.

If $\alpha_B \leq \alpha_S$, we know that banks' investments in quality increase with the level of the interchange fee. Therefore, it is optimal to increase the profit maximising interchange fee above the benchmark interchange fee, which is selected if the level of quality is exogenous. However, the profit maximising interchange fee is already equal to the Acquirer's margin. So the constraint is binding at the equilibrium, and the Acquirer makes no profit.

If $\alpha_B > \alpha_S$, banks' investments in quality can decrease with the level of the interchange fee. So it can be optimal to choose an interchange fee which is lower than the Acquirer's margin, in order to provide both banks with incentives to invest in quality.

The results are quite intuitive. If the Acquirer invests a lot, and if cardholders enjoy more the benefits of a higher level of quality than merchants, the optimal interchange fee is lower, because the investments of the Acquirer and the interchange fee are strategic substitutes. On the contrary, if the Acquirer does not contribute a lot to investments, and if investments in quality are relatively more beneficial for merchants, the interchange fee should be increased, because of strategic complementarity.

Comparison of the profit maximising interchange fee and the social maximising interchange fee. Assume that, at the first stage of the game, a benevolent social planner chooses the interchange fee, a^W, which maximises the social welfare under the budget constraints. My aim is to determine the conditions under which there is an overprovision of payment card services if the payment platform maximises banks' joint profits. I assume that the social welfare is a concave function of the interchange fee a.

Proposition 3 *If $\alpha_B \leq \alpha_S$, the welfare maximising and the profit maximising interchange fee are equal, $a^P = a^W$. If $\alpha_B > \alpha_S$, the welfare maximising interchange fee is lower than the profit maximising interchange fee if and only if $1 + \alpha_B (\theta^*)'(a^P) \leq 0$.*

Proof. See Appendix H. ■

If the strategic effects are positive, the welfare maximising and the profit maximising interchange fee are equal. It would be optimal to increase the interchange fee so as to provide banks with incentives to invest in quality. However, the constraint of positivity on the Acquirer's profit is already binding. So the welfare maximising and the profit maximising interchange fees are equal.

If $1 + \alpha_B(\theta^*)'(a) \leq 0$, there may be an overprovision of payment card services, if the interchange fee is chosen by a profit maximising payment platform, as in Rochet and Tirole's model (2002), but not for the same reasons. This is because banks fail to internalise the strategic impact of their behaviour of their quality choices on the consumers' surplus. If $1 + \alpha_B(\theta^*)'(a) \leq 0$, the surplus of consumers, which depends only on the transaction volume, is a decreasing function of the interchange fee. Since the social welfare takes into account the surplus of consumers, the welfare maximising interchange fee is lower than the profit maximising interchange fee.

Coordination of investments in quality. The payment platform chooses an interchange fee so as to correct two types of externalities. The first type of externality is related to the demand side, and the behaviour of consumers and merchants when they decide whether or not to use and accept cards. The second type of externality is linked to the fact that banks fail to coordinate when they decide on the levels of quality. A natural solution to this coordination problem is to let the payment card platform choose the levels of quality at the first stage, in order to maximise banks' joint profits. In practice, some payment platforms impose on banks a minimum level of investments, and banks coordinate on the choice of common infrastructure standards and processing rules. However, banks remain also free to choose by themselves to improve the level which is fixed by the payment platform. Also, payment cards systems cannot sign contracts on all investment decisions. However, it is interesting to compare the levels of quality and interchange fee obtained in the previous section, to a situation in which the platform would control the levels of investments.

Proposition 4 *If the payment platform controls the levels of quality and the interchange fee, it chooses the maximum level of interchange fee compatible with the budget constraint of the Acquirer, a^*. The levels of quality verify*

$$C_I^{'}(\theta_I) = (\alpha_B + \alpha_S)\lambda_I V(a^*, \theta) + \frac{\alpha_B \lambda_I}{2}(b_S + \alpha_S\theta - a^* - c_A),$$

and

$$C_A^{'}(\theta_A)/\lambda_A = C_I^{'}(\theta_I)/\lambda_I.$$

The welfare maximising interchange fee a^W is also the maximum interchange fee compatible with the budget constraint of the Acquirer. The welfare maximising levels of quality are higher, for a given level of the interchange fee, than the profit maximising levels of quality.

Proof. See Appendix I. ■

The payment system chooses the levels of quality such that the marginal contribution of each bank is equal to the sum of the marginal benefits generated for both banks. Since the quality level of the payment system is a public good, for a given level of the interchange fee, θ_I and θ_A are higher if they are chosen simultaneously by the payment platform, because it internalises the externalities that banks exert on one another. The optimal interchange fee is the highest that ensures non negative profits for the Acquirer.

In general, it is difficult to compare the profit maximising interchange fee if there is cooperation on quality investments, a_C^P, and the interchange fee if banks choose the quality levels non cooperatively, a^P. By studying the special cases where $\alpha_B = 0$, and $\alpha_S = 0$, I show that a_C^P may be either higher or lower than a^P. I also compare the profit maximising interchange fees, a^P and a_C^P, and the welfare maximising interchange fees, a^W and a_C^W, and Baxter's interchange fee $a^B = b_S - c_A$.

Proposition 5 *Depending on the specification of the costs functions, and on the relative impact of quality on each side of the market,* a_C^P *may be higher or lower than* a^P*.*

Proof. See Appendix J. For comparisons in special cases, see Appendix J. ■

4 An example

In this section, I specify the quality cost function to understand better the impact of the strategic interactions between banks on the optimal interchange fee. I assume that $C_i(\theta_i) = (k/2)\theta_i^2$, with $k > 0$ for $i = I, A$. The other assumptions are the same as in the general presentation of the model.[15]

In order to precise the sign of the strategic effects if $\alpha_S < \alpha_B$, I analyse an example in which the level of quality does not affect merchants' willingness to pay for card services ($\alpha_S = 0$). In that case, both banks invest in quality at stage 2, and the equilibrium levels of quality are:

$$\theta_I^*(a) = \frac{\alpha_B \lambda_I}{2k - \alpha_B^2 \lambda_I^2} \left[(1 + a - c_I) + \frac{\alpha_B^2 \lambda_A^2}{2k}(b_S - c_A - a) \right],$$

and

$$\theta_A^*(a) = \frac{\lambda_A \alpha_B}{2k}(b_S - c_A - a),$$

[15] Assumption 3 yields with this specification of the cost functions $2k > (\alpha_B \lambda_I)^2$ and $k > (\alpha_S \lambda_A)^2$. We also assume that $k > \alpha_S \alpha_B \lambda_A^2$. See Appendix K for the proof of the results.

and the quality of the payment card system is

$$\theta^*(a) = \frac{\alpha_B}{2k - \alpha_B^2 \lambda_I^2} \left[\lambda_I^2(1 + a - c_I) + \lambda_A^2(b_S - c_A - a)\right].$$

For the Acquirer, the interchange fee and the investments are strategic substitutes ($\alpha_S = 0 \leq \alpha_B$), while for the Issuer, they are strategic complements. In this example, the strategic effect of the Issuer's quality choice is positive, while the strategic effect of the Acquirer's quality choice is negative.[16] Since $\alpha_S = 0$, the Acquirer does not benefit from quality investments through a higher margin on merchants' side.

At stage one, the payment system chooses the interchange fee a^P which maximises banks' joint profits. The Acquirer's constraint is not binding, and the optimal interchange fee is lower than the Acquirer's margin, and than Baxter's interchange fee (See Appendix K).

In this example, the strategic effect of the Acquirer's quality choice dominates the strategic effect of the Issuer's quality choice. The Acquirer contributes to the increase of the transaction volume by its quality investments, and its level of quality θ_A^* decreases with the interchange fee. Therefore, since this effect dominates the strategic effect of the Issuer's choice of quality, it is optimal for the payment system to choose a smaller interchange fee than in the benchmark case to provide the Acquirer with incentives to invest in quality. In this case, it is optimal to substitute the interchange fee for investments in quality on the Acquisition side.

It is interesting to note that, in this case, if $\lambda_A = 0$, θ^* is higher than the level of quality obtained with coordination, θ^C. This example shows that the payment platform obtains a higher quality in some cases when banks choose the level of quality non cooperatively.

5 Extensions and discussions

In this section, I start by discussing the impact of the structure of the interchange fee. Then, I consider two extensions of the model. First, I discuss the influence of the market structure on the results obtained in section three. Second, I discuss the extension of the analysis to the case of heterogeneous merchants.

5.1 The structure of the interchange fee: fixed fee versus two-part tariff?

My results show that, if the strategic effects are positive, the payment platform chooses the maximum interchange fee compatible with the budget constraint of the Acquirer. Since the

[16] We have $\left(\theta_I^*\right)'(a) = \frac{\alpha_B \lambda_I}{2k - \alpha_B^2 \lambda_I^2} \times \frac{2k - \alpha_B^2 \lambda_A^2}{2k} \geq 0$ and $\left(\theta_A^*\right)'(a) = \frac{-\alpha_B \lambda_A}{2k} < 0$.

constraint is binding, the payment platform could increase banks' joint profits by choosing a fixed transfer T paid by the Issuer to the Acquirer, provided that the Issuer's profit remains positive. In this case, the interchange fee becomes

$$a^P = b_S + \alpha_S(\theta^*)(a^P) - c_A + 2\left[S_I(a^P) + S_A(a^P)\right],$$

and

$$T = -\pi_A(a^P, (\theta_I^*)(a^P), (\theta_A^*)(a^P)).$$

This shows that, unlike Baxter's interchange fee, a two-part tariff may be an appropriate structure for the interchange fee if banks incur fixed costs of investments in quality, and if the Acquirer's budget constraint is binding.

5.2 The impact of the market structure

Like Schmalensee (2002), I assumed that the market is made of a bilateral monopoly, which is generally not the case in the payment card industry. Though several market structures can be observed all over Europe, the concentration is often higher in the issuing industry than in the acquisition business. It is beyond the scope of this paper to determine the determinants of market concentration in the payment card industry, because the market structure is exogenous to the model. However, a few conclusions can be made with my model for other market structures than bilateral monopoly. For instance, assume that the issuer is a monopolist, while the market for acquisition is perfectly competitive. If acquirers make zero profit, they do not invest in quality. So, the solution to the problem is obtained by setting $\lambda_A = 0$ in the model. In this case, $\theta_A^* = 0$ and $(\theta_I^*)'(a) \geq 0$. So the interchange fee remains at the maximum level compatible with merchant acceptance. More generally, the model gives the intuition that if the issuing business is more concentrated than the acquisition market, the interchange fee should remain at the maximum level compatible with merchant acceptance. However, if the acquisition side is more concentrated that the issuing side, the interchange fee might be decreased, because the interchange fee and the Acquirer's investments may become strategic substitutes.

5.3 The impact of merchant heterogeneity

In this subsection, I try to examine how the results of the model are modified if merchants are heterogeneous. I assume that the card acceptance benefit b_S is distributed according to a uniform distribution over $[0, 1]$. Let me denote the percentage of merchants that accept card payments by $D_S(m)$.[17] I assume that, at the equilibrium, some merchants accept cards, but not all; this allows me to study the difference between this situation and the corner solution obtained when merchants are homogeneous. For a given merchant fee m, merchants' demand is $D_S(m) = 1 + \alpha_S\theta - m$, and the transaction volume is expressed as follows:

$$V(\theta, f, m) = (1 + \alpha_S\theta - m)(1 + \alpha_B\theta - f).$$

The general expression of banks' profits is not modified. However, in Appendix J, I show that the levels of quality are not necessarily strategic complements. The strategic complementarity of θ_I and θ_A depends now on the interchange fee.

I start by determining the benchmark interchange fee, when the level of quality is exogenous, then I study the impact of investments in quality on the optimal interchange fee.

Proposition 6 *If the level of quality is exogenous, and if merchants are heterogeneous, the profit maximising interchange fee selected by the payment platform equalises banks' marginal costs net of the marginal benefits of investments, such that:*

$$c_I - a^E(\theta) - \alpha_B\theta = c_A + a^E(\theta) - \alpha_S\theta.$$

Proof. In Appendix L1, I prove that the optimal interchange fee is:

$$a^E(\theta) = \frac{(c_I - \alpha_B\theta) - (c_A - \alpha_S\theta)}{2}.$$

This interchange fee equalises banks' marginal costs net of the marginal benefits of investment, such that:

$$c_I - a^E(\theta) - \alpha_B\theta = c_A + a^E(\theta) - \alpha_S\theta.$$

■

If the levels of quality are exogenous, and if merchants are heterogeneous, the payment system chooses an interchange fee which balances demands between each side of the market, and which may leave some positive margin to the Acquirer. As in Schmalensee (2002), who assumes identical linear demands under bilateral monopoly, the optimal interchange fee is set to equalize banks' marginal costs. However, in the present setting, the marginal costs must be considered as net of the marginal benefits of investments, such that:

$$c_I - a^E(\theta) - \alpha_B\theta = c_A + a^E(\theta) - \alpha_S\theta.$$

[17] The size of the merchants' population is normalised to one.

Proposition 7 *If the levels of quality are endogenous, if merchants are heterogeneous, and if the constraint $\pi_A \geq 0$ is not binding, the optimal interchange fee satisfies:*

$$a^P = a^E((\theta^*)(a^P)) + \frac{4}{2 + (\alpha_S + \alpha_B)(\theta^*)(a) - c} \left(S_I(a^P) + S_A(a^P)\right),$$

where $S_j(a^P)$ denotes the strategic effect for $j = I, A$.

Proof. See Appendix L2. ■

This equation shows that, if the strategic effects are not equal to zero, the optimal interchange fee is no longer set such that banks' marginal costs are equalized, because the marginal costs must be adjusted by the strategic effects:

$$c_I - a^P - \alpha_B\theta - \frac{2\left(S_I(a^P) + S_A(a^P)\right)}{2 + (\alpha_S + \alpha_B)(\theta^*)(a^P) - c} = c_A + a^P - \alpha_S\theta - \frac{2\left(S_I(a^P) + S_A(a^P)\right)}{2 + (\alpha_S + \alpha_B)(\theta^*)(a^P) - c}.$$

The sign and the magnitude of the strategic effects depend on the shape of the best response functions, which could be determined by specifying the cost functions. Under the assumption that $C_i(\theta_i) = (k/2)\theta_i^2$ for $i = I, A$, it is not possible to find a simple solution.

6 Conclusion

My paper extends Baxter (1983)'s model by showing that the level of quality of the payment system influences the choice of the optimal interchange fee. Under bilateral monopoly, if the level of quality is exogenous, the optimal interchange fee is equal to the Acquirer's margin, which depends on the level of quality perceived on the merchant side. However, this is not necessarily the case if banks have the possibility to invest in quality. If investments in quality impact relatively more the consumer side, the optimal interchange fee can be strictly lower than the Acquirer's margin. This proves that a payment system can choose to lower its interchange fee to provide the Acquirer with incentives to invest in a higher level of quality. If merchants are heterogeneous, under bilateral monopoly, my model extends Schmalensee (2002)'s results by showing that the optimal interchange fee equalises bank's marginal costs net of the marginal benefits of investments.

The analysis conducted in my paper should contribute to provide other insights in the current debates about interchange fees. For instance, in Danemark, there were no interchange fees for debit cards. However, in 2003, when banks faced high investment costs to implement the new technical standard "EMV", they discussed the necessity of setting up an interchange fee.[18] This example shows that banks consider the interchange fee as a mechanism which may foster investments to improve the level of quality of a payment system. The results of my paper confirm this view, by showing that the level of quality of the payment system should guide the choice of the optimal interchange fee. This explains also why payment systems in Europe that provide different types of services to cardusers and merchants have chosen different levels of interchange fees. My paper provides another interesting point for the discussions pertaining to the standardisation of payment card services in Europe. It shows that the setting up of a common level of quality for card payments in Europe, which will require different levels of investments across the European countries, should be financed by different pricing policies.

Another interesting topic that is not modelled in my paper is the fact that merchants can also invest themselves to improve the quality of the service perceived by the consumer. This is left to future research.

References

[1] ARMSTRONG M. (2006): "Competition in Two-Sided Markets", The RAND Journal of Economics, vol.37 pp. 668-691.

[2] ARMSTRONG M. & WRIGHT J. (2007): "Two-Sided markets, Competitive Bottlenecks and Exclusive Contracts", Economic Theory, vol. 32, pp. 353-380.

[3] BAXTER W. (1983): "Bank Interchange of Transactional Paper: Legal and Economic Perspectives", Journal of Law & Economics, vol. 26, no. 3, October, pp. 541-588.

[4] CARLTON D.W. AND A.S. FRANKEL (1995): “The antitrust economics of payment card networks”, Antitrust Law Journal, 63:643-668.

[5] CAILLAUD B. & JULLIEN B. (2003): "Chicken and Egg: Competition among Intermediation Service Providers", The RAND Journal of Economics, vol. 34, no. 2, June, pp. 521-552.

[6] CHAKRAVORTI S. (2003): "Theory of Credit Card Networks: A survey of the literature", Review of Network Economics, vol. 2, no. 2, June, pp. 50-68.

[7] CHAKRAVORTI S. & ROSON, R. (2006): "Platform competition in Two-Sided Markets: The Case of Payment Networks", Review of Network Economics, vol. 5, no. 1, March, pp. 26-44.

[18] Source "Banking Automation Bulletin", March 2005.

[8] GANS J. & KING S. (2003): "The Neutrality of Interchange Fees in Payment Systems", Topics in Economic Analysis & Policy, Berkeley Electronic Press, vol. 3, no. 1, pp. 1069-1069.

[9] GUTHRIE G. & WRIGHT J. (2007): "Competing Payment Schemes", Journal of Industrial Economics, vol. 55, pp. 37-67.

[10] ROCHET J-C. (2003): "the Theory of Interchange Fees: A synthesis of recent contributions", Review of Network Economics, vol. 2, no. 2, June, pp. 97-124.

[11] ROCHET J-C. & TIROLE J. (2002): "Cooperation Among Competitors: The Economics of Payment Card Associations", The RAND Journal of Economics, vol. 33, no. 4, winter, pp. 549-570.

[12] ROCHET J-C. & TIROLE J. (2003a): "An Economic Analysis of the Determination of Interchange Fees in Payment Card Systems", Review of Network Economics, vol. 2, no. 2, June, pp. 69-79.

[13] ROCHET J-C. & TIROLE J. (2006a): "Externalities and Regulation in Card Payment Systems", Review of Network Economics, vol. 5, no. 1, March, pp. 1-14.

[14] ROCHET J-C. & TIROLE J. (2006b): "Two-sided markets: a progress report", RAND Journal of Economics, vol. 37, no. 3, March, pp. 645-667.

[15] ROCHET J-C. & TIROLE J. (2007): "Must-Take Cards and the Tourist Test", Mimeo.

[16] SCHMALENSEE R. (2002): "Payment systems and Interchange Fees", Journal of Industrial Economics, vol. 50, no. 2 (June), pp. 103-122.

[17] WRIGHT J. (2002): "Optimal Payment Card Systems," European Economic Review, vol. 47, no. 4, August, pp. 587-612.

[18] WRIGHT J. (2004): "Determinants of Optimal Interchange Fees in Payment Systems", Journal of Industrial Economics, vol. 52, no. 1, March, pp. 1-26.

7 Appendix

Appendix A: proof of Lemma 1. For given θ_A and a, the Issuer chooses the level of quality, θ_I, that maximises its profit,

$$\pi_I(a, \theta_I, \theta_A) = V(a, \theta) M_I(a, \theta) - C_I(\theta_I). \tag{A1}$$

The first-order condition can be written as:[19]

$$\frac{\partial \pi_I}{\partial \theta_I} = \frac{\partial V(a,\theta)}{\partial \theta_I} M_I + \frac{\partial M_I(a,\theta)}{\partial \theta_I} V(a,\theta) - C_I^{'}(\theta_I) = 0. \tag{A2}$$

Since $M_I = V(a,\theta) = (1 + \alpha_B \theta + a - c_I)/2$, then $\partial V(a,\theta)/\partial \theta_I = (\alpha_B \lambda_I)/2$.
Replacing for M_I in (A2), I obtain $\dfrac{\partial \pi_I}{\partial \theta_I} = \alpha_B \lambda_I V(a,\theta) - C_I^{'}(\theta_I) = 0$, which yields $C_I^{'}(\theta_I) = \alpha_B \lambda_I V(a,\theta)$.

The optimal level of quality is chosen by the Issuer such that the marginal cost of investments is equal to the marginal benefits.
For given θ_I and a, the Acquirer chooses the level of quality that maximises its profit,

$$\pi_A(a,\theta_I,\theta_A) = V(a,\theta) M_A(a,\theta) - C_A(\theta_A). \tag{A3}$$

The first-order condition can be written as[20]

$$\frac{\partial \pi_A}{\partial \theta_A} = \frac{\partial M_A(a,\theta)}{\partial \theta_A} V(a,\theta) + \frac{\partial V(a,\theta)}{\partial \theta_A} M_A(a,\theta) - C_A^{'}(\theta_A) = 0. \tag{A4}$$

Since $M_A(a,\theta) = b_S + \alpha_S \theta - a - c_A$, and $\partial M_A(a,\theta)/\partial \theta_A = \alpha_S \lambda_A$, the first order condition becomes

$$V(a,\theta)\alpha_S \lambda_A + \frac{\alpha_B \lambda_A}{2}(b_S + \alpha_S \theta - a - c_A) - C_A^{'}(\theta_A) = 0.$$

Given a, the Acquirer's best response to θ_I satisfies the following equation,

$$C_A^{'}(\theta_A) = \alpha_S \lambda_A V(a,\theta) + \frac{\alpha_B \lambda_A}{2}(b_S + \alpha_S \theta - a - c_A).$$

The optimal level of quality is chosen by the Acquirer such that the marginal cost of investments is equal to the marginal benefits.

Appendix B: proof of Lemma 2. I prove the strategic complementarity of the levels of quality. I differentiate the first-order condition obtained from (A4) with respect to θ_I :

$$\frac{d}{d\theta_I}\left(\frac{\partial \pi_A}{\partial \theta_A}(a,\theta_I,R_A(a,\theta_I))\right) = 0,$$

which yields:

$$\frac{\partial^2 \pi_A}{\partial \theta_I \partial \theta_A} + \frac{\partial^2 \pi_A}{\partial {\theta_A}^2} \times \frac{\partial R_A(a,\theta_I)}{\partial \theta_I} = 0.$$

[19] Assumption A3 implies that the second-order condition is satisfied.
[20] Assumption A3 implies that the second-order condition is satisfied.

I already know from the second-order condition that:

$$\frac{\partial^2 \pi_A(a,\theta)}{(\partial \theta_A)^2} < 0, \tag{B1}$$

therefore:

$$\frac{\partial R_A(a,\theta_I)}{\partial \theta_I} = -\left(\frac{\partial^2 \pi_A}{\partial {\theta_A}^2}\right)^{-1} \frac{\partial^2 \pi_A}{\partial \theta_I \partial \theta_A}. \tag{B2}$$

I take the partial derivative of (A4) with respect to θ_I:

$$\frac{\partial^2 \pi_A(a,\theta)}{\partial \theta_I \partial \theta_A} = \frac{\partial V(a,\theta)}{\partial \theta_I} \alpha_S \lambda_A + \frac{\alpha_B \lambda_A}{2} \times \frac{\partial M_A(a,\theta)}{\partial \theta_I}.$$

Since $V(a,\theta) = (1 + \alpha_B \theta + a - c_I)/2$, then $\partial V(a,\theta)/\partial \theta_I = (\alpha_B \lambda_I)/2$.

Since $M_A(a,\theta) = b_S + \alpha_S \theta - a - c_A$, then $\partial M_A(a,\theta)/\partial \theta_I = \alpha_S \lambda_I$.

I conclude that $\dfrac{\partial^2 \pi_A(a,\theta)}{\partial \theta_I \partial \theta_A} = \alpha_B \lambda_I \alpha_S \lambda_A \geq 0$.

From (B1) and (B2), I conclude that

$$\frac{\partial R_A(a,\theta_I)}{\partial \theta_I} \geq 0. \tag{B3}$$

The same reasoning can be done for the Issuer's best response:

$$\frac{\partial R_I(a,\theta_A)}{\partial \theta_A} = -\left(\frac{\partial^2 \pi_I}{\partial {\theta_I}^2}\right)^{-1} \frac{\partial^2 \pi_I}{\partial \theta_A \partial \theta_I} \tag{B4}$$

where:

$$\frac{\partial^2 \pi_I}{\partial {\theta_I}^2} < 0, \tag{B5}$$

I differentiate the first-order condition obtained from (A2) with respect to θ_A.

Since $V(a,\theta) = M_I(a,\theta) = (1 + \alpha_B \theta + a - c_I)/2$, then $\partial V(a,\theta)/\partial \theta_A = \partial M_I(a,\theta)/\partial \theta_A = (\alpha_B \lambda_A)/2$.

Therefore, I have $\dfrac{\partial^2 \pi_I}{\partial \theta_A \partial \theta_I} = \dfrac{\alpha_B^2 \lambda_I \lambda_A}{2} \geq 0$.

From (B4) and (B5), I conclude that $\dfrac{\partial R_I(a,\theta_A)}{\partial \theta_A} \geq 0$. Therefore, θ_I and θ_A are strategic complements.

Appendix C: proof of Lemma 3. I determine whether the levels of quality and the interchange fee are strategic complements or strategic substitutes.

I differentiate the first-order condition obtained from (10) with respect to a:

$$\frac{d}{da}\left(\frac{\partial \pi_A}{\partial \theta_A}(a, \theta_I, R_A(a,\theta_I))\right) = 0,$$

which yields

$$\frac{\partial^2 \pi_A(a,\theta)}{\partial a \partial \theta_A} + \frac{\partial^2 \pi_A(a,\theta)}{(\partial \theta_A)^2} \times \frac{\partial R_A(a,\theta_I)}{\partial a} = 0. \tag{C1}$$

I already know from the second-order condition that $\frac{\partial^2 \pi_A(a,\theta)}{(\partial \theta_A)^2} < 0$. Equation (C1) becomes:

$$\frac{\partial R_A(a,\theta_I)}{\partial a} = -\left(\frac{\partial^2 \pi_A(a,\theta)}{(\partial \theta_A)^2}\right)^{-1} \frac{\partial^2 \pi_A(a,\theta)}{\partial a \partial \theta_A}. \tag{C2}$$

Therefore, $\partial R_A(a,\theta_I)/\partial a$ has the same sign as $\partial^2 \pi_A(a,\theta)/\partial a \partial \theta_A$.

I take the partial derivative of (A4) with respect to a :

$$\frac{\partial^2 \pi_A(a,\theta)}{\partial a \partial \theta_A} = \frac{\partial M_A}{\partial \theta_A}\frac{\partial V(a,\theta)}{\partial a} + \frac{\partial M_A}{\partial a}\frac{\partial V(a,\theta)}{\partial \theta_A}.$$

Since $V(a,\theta) = (1 + \alpha_B \theta + a - c_I)/2$, then $\partial V(a,\theta)/\partial a = 1/2$ and $\partial V(a,\theta)/\partial \theta_A = (\alpha_B \lambda_A)/2$.
Since $M_A(a,\theta) = b_S + \alpha_S \theta - a - c_A$, then $\partial M_A(a,\theta)/\partial \theta_A = \alpha_S \lambda_A$ and $\partial M_A(a,\theta)/\partial a = -1$.
Therefore, I have $\partial^2 \pi_A(a,\theta)/\partial a \partial \theta_A = \lambda_A(\alpha_S - \alpha_B)/2$.

If $\alpha_B \leq \alpha_S$, then $\frac{\partial^2 \pi_A(a,\theta)}{\partial a \partial \theta_A} \geq 0$, which implies , from (C2), that $\frac{\partial R_A(a,\theta_I)}{\partial a} \geq 0$.
Therefore, if $\alpha_B \leq \alpha_S$, then θ_A and a are strategic complements.
If $\alpha_B > \alpha_S$, then θ_A and a are strategic substitutes.

Similarly, for the Issuer, the sign of $\partial R_I(a,\theta_A)/\partial a \geq 0$ is the same as the sign of $\partial^2 \pi_I(a,\theta)/\partial a \partial \theta_I$.
I take the partial derivative of (A2) with respect to a, that is $\frac{\partial^2 \pi_I(a,\theta)}{\partial a \partial \theta_I} = \frac{\alpha_B \lambda_I}{2} \geq 0$.

Therefore, I have $\frac{\partial R_I(a,\theta_A)}{\partial a} \geq 0$. I conclude that θ_I and a are strategic complements.

Appendix D: proof of Lemma 4. I determine how the optimal levels of quality vary with the interchange fee.
Let us differentiate (12) with respect to a:

$$\left(\theta_I^*\right)'(a) = \frac{\partial R_I}{\partial a} + \frac{\partial R_I}{\partial \theta_A} \times \frac{\partial R_A}{\partial a}.$$

If $\alpha_B \leq \alpha_S$, from Lemma 3, I have $\partial R_A(a,\theta_I)/\partial a \geq 0$ and $\partial R_I(a,\theta_A)/\partial a \geq 0$.
Since, from Lemma 2, $\partial R_I/\partial \theta_A \geq 0$, I conclude that $\left(\theta_I^*\right)'(a) \geq 0$.
If $\alpha_B > \alpha_S$, then, from Lemma 3, $\partial R_A(a,\theta_I)/\partial a \leq 0$ and $\partial R_I(a,\theta_A)/\partial a \geq 0$. Since, from Lemma 2, $\partial R_I/\partial \theta_A \geq 0$, the sign of $\theta_I'(a)$ can be either positive or negative.
The same reasoning can be applied for the Acquirer, differentiating (11) with respect to a:

$$\left(\theta_A^*\right)'(a) = \frac{\partial R_A}{\partial a} + \frac{\partial R_A}{\partial \theta_I} \times \frac{\partial R_I}{\partial a}.$$

If $\alpha_B \leq \alpha_S$, then, from Lemma 3, $R_A(a,\theta_I)/\partial a \geq 0$ and $\partial R_I(a,\theta_A)/\partial a \geq 0$. Since, from Lemma 2, $\partial R_A(a,\theta_I)/\partial\theta_I \geq 0$, I conclude that $(\theta_A^*)'(a) \geq 0$.

If $\alpha_B > \alpha_S$, then, from Lemma 3, $\partial R_A(a,\theta_I)/\partial a \leq 0$ and $\partial R_I(a,\theta_A)/\partial a \geq 0$. Since, from Lemma 2, $\partial R_A/\partial\theta_I \geq 0$, the sign of $\theta_A^{'}(a)$ can be either positive or negative.

Appendix E: proof of Proposition 1. I determine the optimal interchange fee if the level of quality is exogenous. The differentiation of banks' profits with respect to a yields:

$$\frac{d\pi_I^E(a)}{da} = V_I^E(a) = \frac{1+\alpha_B\theta + a - c_I}{2}. \tag{E1}$$

and

$$\frac{d\pi_A^E(a)}{da} = \frac{1}{2}M_A^E(a) - V_I^E(a) = \frac{-1 + b_S + (\alpha_S - \alpha_B)\theta - 2a + c_I - c_A}{2}. \tag{E2}$$

I write the first-order condition for the maximisation of banks' joint profits,[21] using (E2) and (E1), that is

$$\frac{d\pi_S^E(a)}{da} = \frac{b_S - c_A + \alpha_S\theta - a}{2} = 0.$$

The optimal interchange fee is

$$a^E = b_S - c_A + \alpha_S\theta.$$

The Issuer's profit is given by:

$$\pi_I^E = \left(V^E\right)^2 = \frac{1}{4}(1 + b_S + (\alpha_B + \alpha_S)\theta - c)^2.$$

I know from (4) that the Acquirer chooses the merchant fee m such that $m(\theta) = b_S + \alpha_S\theta$. Therefore, merchants make no surplus. The social welfare, W, is expressed as follows:

$$W = \pi_S + S_C,$$

where the average consumer surplus S_C is[22]:

$$S_C = \int_{f-\alpha_B\theta}^{1} (b_B + \alpha_B\theta - f)\, db_B = \frac{(1+\alpha_B\theta - f)^2}{2}. \tag{E3}$$

[21] The second-order condition is verified.

[22] A consumer of type b_B chooses to use his card if and only if (2) holds, and he makes the surplus $b_B + \alpha_B\theta - f$. Recall that we assumed that b_B is uniformely distributed over $[0;1]$.

The social welfare is expressed as follows:

$$W^E(a) = \pi_S^E(a) + \frac{1}{2}(V^E(a))^2.$$

I compute the derivative of social welfare with respect to a:

$$\frac{dW^E(a)}{da} = \frac{d\pi_S^E(a)}{da} + \frac{dV^E(a)}{da}V^E(a) = \frac{2b_S + 1 + (2\alpha_S + \alpha_B)\theta - 2c_A - c_I - a}{2}, \tag{E4}$$

The objective function of the social planner is concave.
Since $V^E(a) = (1 + \alpha_B\theta + a - c_I)/2$, then $\dfrac{dV^E(a)}{da}V^E(a) \geq 0$.
Therefore, from (E4), I have $\dfrac{dW^E(a)}{da} \geq \dfrac{d\pi_S^E(a)}{da}$.
The following inequality holds $\left.\dfrac{dW^E(a)}{da}\right|_{a=a^E(\theta)} \geq \left.\dfrac{d\pi_S^E(a)}{da}\right|_{a=a^E(\theta)}$. Since $\left.\dfrac{d\pi_S^E(a)}{da}\right|_{a=a^E(\theta)} = 0$, $\left.\dfrac{dW^E(a)}{da}\right|_{a=a^E(\theta)} \geq 0$.
Let us denote by a^W the welfare maximising interchange fee. It verifies the first-order condition $\left.\dfrac{dW^E(a)}{da}\right|_{a=a^W} = 0$.
Therefore, I have $\left.\dfrac{dW^E(a)}{da}\right|_{a=a^E(\theta)} \geq \left.\dfrac{dW^E(a)}{da}\right|_{a=a^W}$. Since W is concave, I conclude that $a^E(\theta) \leq a^W$.
The welfare maximising interchange fee cannot be lower than the profit maximising interchange fee. However, the profit maximising interchange fee is already set at the maximum level compatible with a positive margin for the Acquirer. So, the constraint is binding at the equilibrium, and the welfare maximising interchange fee is equal to the profit maximising interchange fee.
Replacing for $a^E(\theta)$ in (E3), I get the expression of the social welfare at the welfare maximising interchange fee:

$$W^E = \frac{3}{8}(1 + b_S + (\alpha_B + \alpha_S)\theta - c)^2.$$

Appendix F: Strategic effects. I determine the sign of the strategic effects. I differentiate the Issuer's profit with respect to a, which yields:

$$\frac{d\pi_I}{da} = \frac{\partial\pi_I}{\partial a} + \frac{\partial\pi_I}{\partial\theta_I}(\theta_I^*)'(a) + \frac{\partial\pi_I}{\partial\theta_A}(\theta_A^*)'(a).$$

The application of the envelop theorem yields $\dfrac{\partial\pi_I}{\partial\theta_I}(a, \theta_I^*(a), \theta_A^*(a)) = 0$.
Therefore, $\dfrac{d\pi_I}{da} = \dfrac{\partial\pi_I}{\partial a} + \dfrac{\partial\pi_I}{\partial\theta_A}(\theta_A^*)'(a)$. From (A1), I know that $\dfrac{\partial\pi_I}{\partial a} = \dfrac{\partial M_I}{\partial a}V + \dfrac{\partial V}{\partial a}M_I$, and that $\dfrac{\partial\pi_I}{\partial\theta_A} = \dfrac{\partial M_I}{\partial\theta_A}V + \dfrac{\partial V}{\partial\theta_A}M_I$. From (7), I find that $\partial M_I/\partial a = 1/2$, and that $\partial M_I/\partial\theta_A = (\alpha_B\lambda_A)/2$.

From (6), I find that $\partial V/\partial a = 1/2$, and that $\partial V/\partial \theta_A = \dfrac{\alpha_B \lambda_A}{2}$. Since $M_I = V$, I have:

$$\frac{d\pi_I}{da} = V(a, \theta^*(a)) + \alpha_B \lambda_A (\theta_A^*)'(a) V^{II}(a, \theta^*(a)). \tag{F1}$$

As a result, the direct effect is positive for the Issuer, and the strategic effect has the same sign as $(\theta_A^*)'(a)$.

The same reasonning and the application of the envelop theorem to the Acquirer's profit yields $\dfrac{d\pi_A}{da} = \dfrac{\partial \pi_A}{\partial a} + \dfrac{\partial \pi_A}{\partial \theta_I}(\theta_I^*)'(a)$. From (A3), I know that $\dfrac{\partial \pi_A}{\partial a} = \dfrac{\partial M_A}{\partial a} V + \dfrac{\partial V}{\partial a} M_A$, and that $\dfrac{\partial \pi_A}{\partial \theta_I} = \dfrac{\partial M_A}{\partial \theta_I} V + \dfrac{\partial V}{\partial \theta_I} M_A$. From (8), I know that $\partial M_A/\partial a = -1$, and that $\partial M_A/\partial \theta_I = \alpha_S \lambda_I$. From (6), I know that $\partial V/\partial \theta_I = (\alpha_B \lambda_I)/2$, so

$$\frac{d\pi_A}{da} = \frac{1}{2} M_A - V(a, \theta^*(a)) + \left[\alpha_S \lambda_I V(a, \theta^*(a)) + \frac{\alpha_B \lambda_I}{2} M_A\right] (\theta_I^*)'(a). \tag{F2}$$

Consequently, the direct effect may be positive or negative for the Acquirer, and the strategic effect has the same sign as $(\theta_I^*)'(a)$.

Appendix G: proof of Proposition 2. Let us denote the strategic effects by $S_i(a)$ for bank i for $i = \{I; A\}$. I have $S_i(a) = \dfrac{\partial \pi_i}{\partial \theta_j}(\theta_j^*)'(a)$ for $(i; j) \in \{I; A\}^2$ and $i \neq j$.

The derivative of the payment system profit π_S with respect to the interchange fee a is:

$$\frac{d\pi_S}{da} = \frac{\partial}{\partial a}\left(V(a, \theta) M_S(a, \theta)\right) + S_I(a) + S_A(a). \tag{G1}$$

The result stems directly from (F2) and (F1) and Appendix F. With the parameters of the model, the equation (G1) becomes:

$$\frac{d\pi_S}{da} = \frac{1}{2} M_A(a, \theta) + \left[\frac{\alpha_B \lambda_I}{2} M_A(a, \theta^*(a)) + \alpha_S \lambda_I V(a, \theta^*(a))\right] (\theta_I^*)'(a) + \alpha_B \lambda_A (\theta_A^*)'(a) V(a, \theta^*(a)).$$

If the constraint ($\pi_A \geq 0$) is binding, then the optimal interchange fee, a^P, satisfies the following equation:

$$M_A(a^P, \theta^*(a^P)) V(a^P, \theta^*(a^P)) = C_A(\theta_A^*(a^P)).$$

If the constraint $\pi_A \geq 0$ is not binding, then the optimal interchange fee, a^P, satisfies the first order condition of joint profit maximisation:

$$\frac{d\pi_S}{da} = \frac{1}{2} M_A(a, \theta^*(a)) + S_I(a) + S_A(a) = 0, \tag{G2}$$

which can be written

$$\begin{aligned} S_I(a^P) + S_A(a^P) &= \frac{1}{2}(a^P - b_S - \alpha_S \theta^*(a^P) + c_A), \\ S_I(a^P) + S_A(a^P) &= \frac{1}{2}(a^P - a^E(\theta^*(a^P)). \end{aligned}$$

If $\alpha_B \leq \alpha_S$, from Appendix F and Lemma 4, I know that the strategic effects are positive. If the constraint $\pi_A \geq 0$ were not binding, then from (G2), the profit maximising interchange fee would be higher than the Acquirer's margin, which is absurd. Therefore, if $\alpha_B \leq \alpha_S$, the constraint $\pi_A \geq 0$ is binding, and the optimal interchange fee verifies $\pi_A(a^P) = 0$, which proves the first part of the proposition.

Notice that a^P is the highest interchange fee that satisfies $\pi_A(a^P, \theta_I^*(a^P), \theta_A^*(a^P)) \geq 0$, because π_A is locally decreasing in the neighborhood of a^P:

$$\begin{aligned} \left.\frac{d\pi_A}{da}\right|_{a=a^P} &= \frac{1}{2}M_A(a^P) - V(a^P, \theta^*(a^P)) + S_A(a^P) \\ &= -S_I(a^P) - V(a^P, \theta^*(a^P)) \leq 0. \end{aligned}$$

If the strategic effects are negative, which, according to Lemma 4, may happen only if $\alpha_B > \alpha_S$, and if the constraint $\pi_A \geq 0$ is not binding, the profit maximising interchange fee satisfies the first order condition. Therefore, I have $S_I(a^P) + S_A(a^P) = (a^P - a^E(\theta^*(a^P))/2$, and $a^P - a^E(\theta^*(a^P) < 0$.

Appendix H: proof of Proposition 3. From (E3), the social welfare at the equilibrium of stage 2 is $W = \pi_S(a, \theta_I^*(a), \theta_A^*(a)) + (V(a, \theta^*(a)))^2/2$. I have

$$\left.\frac{dW}{da}\right|_{a=a^P} = \left.\frac{d\pi_S}{da}\right|_{a=a^P} + \left.\frac{dV(a, \theta^*(a))}{da}\right|_{a=a^P} \times V(a^P, \theta^*(a^P)). \tag{H1}$$

The profit maximising interchange fee satisfies the first-order condition, that is $\left.\frac{d\pi_S}{da}\right|_{a=a^P} = 0$. Therefore, from (H1), if $\left.\frac{dV(a, \theta^*(a))}{da}\right|_{a=a^P} \leq 0$, then $\left.\frac{dW}{da}\right|_{a=a^P} \leq \left.\frac{d\pi_S}{da}\right|_{a=a^P}$.

This implies that $\left.\frac{dW}{da}\right|_{a=a^P} \leq 0$. Since $\left.\frac{dW}{da}\right|_{a=a^W} = 0$, this proves that $\left.\frac{dW}{da}\right|_{a=a^P} \leq \left.\frac{dW}{da}\right|_{a=a^W}$.

Since W is concave, if $\left.\frac{dV(a, \theta^*(a))}{da}\right|_{a=a^P} \leq 0$, then $a^P \geq a^W$.

I precise the conditions that imply $\left.\frac{dV(a, \theta^*(a))}{da}\right|_{a=a^P} \leq 0$. The total differentiation of V

with respect to a yields

$$\begin{aligned}\frac{dV(a,\theta^*(a))}{da} &= \frac{\partial V(a,\theta^*(a))}{\partial a} + \frac{\partial V(a,\theta^*(a))}{\partial \theta_I}(\theta_I^*)'(a) + \frac{\partial V(a,\theta^*(a))}{\partial \theta_A}(\theta_A^*)'(a), \\ &= \frac{1}{2} + \frac{\alpha_B \lambda_I}{2}(\theta_I^*)'(a) + \frac{\alpha_B \lambda_A}{2}(\theta_A^*)'(a).\end{aligned}$$

If $\alpha_B \leq \alpha_S$, I know from Lemma 4 that $(\theta_I^*)'(a) \geq 0$ and $(\theta_A^*)'(a) \geq 0$. So, if the strategic effects are positive, the transaction volume is increasing with the interchange fee, and the welfare maximising interchange fee cannot be lower than the profit maximising interchange fee. From Proposition 2, we know that, if $\alpha_B \leq \alpha_S$, the profit maximising interchange fee is set at the maximum level compatible with non negative profits for the Acquirer. So the constraint is binding at the equilibrium, and the welfare maximising interchange fee is equal to the profit maximising interchange fee.

If $\alpha_B > \alpha_S$, then $\left.\frac{dV(a,\theta^*(a))}{da}\right|_{a=a^P} \leq 0$ if and only if $1 + \alpha_B(\theta^*)'(a^P) \leq 0$, which proves the result of the Proposition.

Appendix I: Proof of Proposition 4 In the following proof, I denote by a_C^P the profit maximising interchange fee if the level of quality is chosen by the payment platform, and a_C^W the welfare maximising interchange fee. The payment plaform chooses θ_I, θ_A, and a, in order to maximise banks' joint profits, that is

$$\pi_I + \pi_A = (b_S + \alpha_S\theta - c + \frac{1 + \alpha_B\theta - a + c_I}{2})V(a,\theta) - C_I(\theta_I) - C_A(\theta_A),$$

under the constraints $\pi_I \geq 0$ and $\pi_A \geq 0$. Solving for the first-order condition yields

$$C_I'(\theta_I) = (\alpha_B + \alpha_S)\lambda_I V(a,\theta) + \frac{\alpha_B\lambda_I}{2}(b_S + \alpha_S\theta - a - c_A), \tag{I1}$$

and

$$C_A'(\theta_A) = (\alpha_B + \alpha_S)\lambda_A V(a,\theta) + \frac{\alpha_B\lambda_A}{2}(b_S + \alpha_S\theta - a - c_A). \tag{I2}$$

Since $\frac{d(\pi_A + \pi_I)}{da} = \frac{1}{2}(b_S + \alpha_S\theta - a - c_A)$, the interchange fee is the highest compatible with non negative profit for the Acquirer (otherwise, the interchange fee would be equal to the margin of the Acquirer, and its profit would be negative because of the cost of the investment in quality).

I denote by θ_I^C and θ_A^C the levels of quality obtained if they are chosen by the payment platform. I find that $C_I'(\theta_I^C) \geq C_I'((\theta_I^*)(a))$, and that $C_I'(\theta_A^C) \geq C_I'((\theta_A^*)(a))$. Therefore, the levels of quality are higher if the payment platform internalises the coordination problem faced by the banks.

If the interchange fee is chosen to maximise the social welfare, $W = \pi_I + \pi_A + \frac{1}{2}V(a,\theta)^2$, solving for the first-order condition yields

$$C_I^{'}(\theta_I) = (\frac{3}{2}\alpha_B + \alpha_S)\lambda_I V(a,\theta) + \frac{\alpha_B\lambda_I}{2}(b_S + \alpha_S\theta - a - c_A),$$

and

$$C_A^{'}(\theta_A) = (\frac{3}{2}\alpha_B + \alpha_S)\lambda_A V(a,\theta) + \frac{\alpha_B\lambda_A}{2}(b_S + \alpha_S\theta - a - c_A).$$

Since $\frac{dW}{da} = \frac{1}{2}(b_S + \alpha_S\theta - c_A - a + \frac{1+\alpha_B\theta + a - c_I}{2})$, if the payment platform or the social planner chooses the interchange fee that solves the first-order condition, the budget constraint of the Acquirer is binding. Therefore, the welfare maximising interchange fee is the maximum interchange fee that satisfies the budget constraint of the Acquirer.

Appendix J: Proof of Proposition 5. To prove that a_C^P may be higher or lower than a^P, I study different cases. I show that:

- - If $\alpha_B = 0$, $a^B \leq a_{NC}^P = a_{NC}^W$ and $a^B \leq a_C^P = a_C^W$.
  - If $\alpha_B = 0$, and if $b_S - c_A - 1 + c_I + \alpha_S\lambda_A(C_A^{'})^{-1}(\alpha_S\lambda_A(1-c_I)/2) \leq 0$, then $a_{NC}^P \leq a_C^P$ and $\theta^{NC}(a_{NC}^P) \leq \theta^C(a_C^P)$.
  - If $\alpha_S = 0$, $a^B \geq a_{NC}^P$, and $a^B \geq a_C^P$.
  - If $\alpha_S = 0$, and if $1 + \alpha_B\lambda_I(\theta_I^*)^{'}(a_C^P)/2 \leq 0$, $a_{NC}^P \geq a_C^P$.

Let us start by the case $\alpha_S = 0$. From (G2), I know that, if banks cannot coordinate on the choice of the level of quality, we have

$$\begin{aligned} \frac{d\pi_S}{da} &= \frac{1}{2}M_A(a,\theta) + \left[\frac{\alpha_B\lambda_I}{2}M_A(a,\theta^*(a))\right](\theta_I^*)'(a) + \alpha_B\lambda_A(\theta_A^*)'(a)V(a,\theta^*(a)) \\ &= \frac{1}{2}(b_S - a - c_A)(\frac{\alpha_B\lambda_I}{2}(\theta_I^*)'(a) + 1) + \frac{1}{2}\alpha_B\lambda_A(\theta_A^*)'(a)(1 + \alpha_B(\theta^*)(a)). \end{aligned}$$

Replacing for $a = a^B = b_S - c_A$, I obtain $\left.\frac{d\pi_S}{da}\right|_{a=a^B} = \frac{1}{2}\alpha_B\lambda_A(\theta_A^*)'(a^B)(1 + \alpha_B(\theta^*)(a^B))$.

If $\alpha_S = 0$, from (B2), I obtain the level of quality chosen by the Acquirer if banks cannot cooperate on the quality,

$$(\theta_A^*)(a) = (C_A^{'})^{-1}(\frac{\alpha_B\lambda_A}{2}(b_S - c_A - a)).$$

So $(\theta_A^*)'(a) \leq 0$, and $\left.\frac{d\pi_S}{da}\right|_{a=a^B} \leq 0$. Since I assumed that the second-order conditions are verified, and since $a = a^P$ solves the first-order condition, I obtain the inequality $a^B \geq a^P$. Replacing for $a = a_C^P$ in (G1), I obtain

$$\left.\frac{d\pi_S}{da}\right|_{a=a_C^P} = \frac{1}{2}(b_S - a_C^P - c_A)(\frac{\alpha_B\lambda_I}{2}(\theta_I^*)'(a_C^P) + 1) + \frac{1}{2}\alpha_B\lambda_A(\theta_A^*)'(a_C^P)(1 + \alpha_B(\theta^*)(a_C^P)).$$

Since $(b_S - a_C^P - c_A) = C_A(\theta_A^C)/V$, I find that $b_S - a_C^P - c_A \geq 0$. So, $a^B \geq a_C^P$.

Since $\alpha_B\lambda_A(\theta_A^*)'(a_C^P)(1 + \alpha_B(\theta^*)(a_C^P)) \leq 0$, it is sufficient to have $\alpha_B\lambda_I(\theta_I^*)'(a_C^P)/2 + 1 \leq 0$, to have $\left.\frac{d\pi_S}{da}\right|_{a=a_C^P} \leq 0$. In this case, $a_C^P \geq a^P$. Otherwise a_C^P may be higher or lower than a^P. It is not simple to compare in this case $\theta_A^*(a^P)$ and $\theta_A^C(a_C^P)$.

I now study the case $\alpha_B = 0$. From (I1) and (I2), I obtain

$$C_I^{'}(\theta_I^C) = \lambda_I C_A^{'}(\theta_A^C)/\lambda_A,$$

and

$$\theta_A^C = (C_A^{'})^{-1}(\alpha_S\lambda_A(\frac{1 + a - c_I}{2})).$$

Let $h(a) = \theta_A^C(a) = (C_A^{'})^{-1}(\alpha_S\lambda_A(\frac{1 + a - c_I}{2}))$, and $\theta_I^C = g(\theta_A^C) = (C_I^{'})^{-1}\left(\frac{\lambda_I}{\lambda_A}C_A^{'}(\theta_A^C)\right)$.

The profit maximising interchange fee with coordination on the choices of the quality levels satisfies to the budget constraint of the Acquirer, that is

$$(b_S + \alpha_S\theta^C(a_C^P) - a_C^P - c_A)\left(\frac{1 + a_C^P - c_I}{2}\right) = C_A(\theta_A^C(a_C^P)),$$

with $\theta^C(a) = \lambda_I \times g(\theta_A^C(a)) + \lambda_A\theta_A^C(a) = \lambda_I \times g(h(a)) + \lambda_A h(a)$.

From (9) and (10), I know, that, if $\alpha_B = 0$, then $\theta_I^*(a) = 0$, and $\theta_A^*(a) = h(a)$. Also, according to Proposition 2, the profit maximising interchange fee is the highest that satisfies to the budget constraint of the Acquirer, that is $(b_S + \alpha_S\theta^*(a^P) - a^P - c_A)\frac{1 + a^P - c_I}{2} = C_A(\theta_A(a^P))$, with $\theta^*(a) = \lambda_A\theta_A^*(a) = \lambda_A h(a)$.

Let us define $x(a) = (b_S + \alpha_S\lambda_A h(a) - a - c_A)(1 + a - c_I)/2 - C_A(h(a))$.

I have $x(a^P) = 0$, and $x(a_C^P) = -\alpha_S\lambda_I \times g(h(a_C^P))\left(\frac{1 + a_C^P - c_I}{2}\right) \leq 0$.

Since $x(a)$ is twice differentiable over $[c_I - 1; 1]$, $x^{'}(a) = \frac{1}{2}(b_S + \alpha_S\lambda_A h(a) - 2a - 1 + c_I - c_A)$, and $x^{"}(a) = \frac{(\alpha_S\lambda_A/2)^2 - C_A^{"}(h(a))}{C_A^{"}(h(a))}$. From assumption 3, we have $x^{"}(a) \leq 0$. So $x^{'}(a)$ is decreasing over $[c_I - 1; 1]$. If $x^{'}(0) \leq 0$, then x is decreasing over $[0; 1]$. Since $\left(a^P; a_C^P\right) \in [0; 1]^2$, and $x(a_C^P) \leq x(a^P)$, this implies that $a_C^P \leq a^P$.

I have $\theta_A^*(a) = h(a) = \theta_A^C(a)$. Since h is increasing with a, if $x^{'}(0) \leq 0$, then $\theta_A^*(a^P) \leq \theta_A^C(a_C^P)$. Since $\theta_I^*(a) = 0$, this implies $\theta^*(a^P) \leq \theta^C(a_C^P)$.

Appendix K: an example. Replacing $C_i^{'}(\theta_i)$ for $k\theta_i$ into (9) and (10) yields to the best response functions for the Acquirer and the Issuer,

$$\theta_A = \frac{\lambda_A}{2(k - \alpha_S\alpha_B\lambda_A^2)}\left(\alpha_B(b_S - c_A - a) + \alpha_S(1 + a - c_I) + 2\alpha_S\alpha_B\lambda_I\theta_I\right),$$

and

$$\theta_I = \frac{\alpha_B\lambda_I}{2k - \alpha_B^2\lambda_I^2}(1 + \alpha_B\lambda_A\theta_A + a - c_I).$$

The properties of the best response functions prooved in Lemma 1 and 2 are verified: qualities are strategic complements, θ_I and a are strategic complements, and θ_A and a are strategic complements if $\alpha_B \leq \alpha_S$.

I compute the Nash Equilibrium of stage 2 by taking the intercept of the best response functions. To simplify the expression of the results, I define $u = b_S - c_A - a$ and $v = 1 + a - c_I$. Equilibrium levels of quality are

$$\theta_I^*(a) = \frac{\lambda_I\alpha_B\left[\left(2k - \alpha_S\alpha_B\lambda_A^2\right)v + \alpha_B^2\lambda_A^2 u\right]}{2(2k - \alpha_B^2\lambda_I^2)(k - \alpha_S\alpha_B\lambda_A^2) - 2\alpha_S\alpha_B^3\lambda_I^2\lambda_A^2}, \tag{I1}$$

$$\theta_A^*(a) = \frac{\lambda_A\left[\left(2k - \alpha_B^2\lambda_I^2 + 2\alpha_B\alpha_S\lambda_I^2\right)\alpha_B u + (2k - \alpha_B^2\lambda_I^2)\alpha_S v\right]}{2(2k - \alpha_B^2\lambda_I^2)(k - \alpha_S\alpha_B\lambda_A^2) - 2\alpha_S\alpha_B^3\lambda_I^2\lambda_A^2}, \tag{I2}$$

and

$$\theta^*(a) = \frac{2\alpha_B u\lambda_A^2(k + \alpha_S\alpha_B\lambda_I^2) + (2k(\alpha_B\lambda_I^2 + 2\lambda_A^2\alpha_S) - 3\alpha_S\lambda_A^2\alpha_B^2\lambda_I^2)v}{2(2k - \alpha_B^2\lambda_I^2)(k - \alpha_S\alpha_B\lambda_A^2) - 2\alpha_S\alpha_B^3\lambda_I^2\lambda_A^2}. \tag{I3}$$

The transaction volume is equal to:

$$V(a) = \frac{u \times 2\alpha_B^2\lambda_A^2(k + \alpha_S\alpha_B\lambda_I^2) + v \times \left(4k^2 - \alpha_S\alpha_B^3\lambda_A^2\lambda_I^2\right)}{2(2k - \alpha_B^2\lambda_I^2)(k - \alpha_S\alpha_B\lambda_A^2) - 2\alpha_S\alpha_B^3\lambda_I^2\lambda_A^2}.$$

If $\alpha_S = 0$, the optimal interchange fee is

$$a_{NC}^P = b_S - c_A - \frac{(2k)(\alpha_B^2\lambda_A^2) \times (1 + b_S - c)}{(2k - \alpha_B^2\lambda_A^2)(2k + \alpha_B^2\lambda_A^2) + \left(2k - \lambda_I^2\alpha_B^2\right)\alpha_B^2\lambda_A^2}.$$

Then I study the case of coordination on the choice of quality levels. With this specification of the cost function, the levels of quality chosen by the payment platform are independent from this interchange fee, and for $i = I; A$,

$$\theta_i^C = \frac{\alpha_B \lambda_i (1 + b_S - c)}{4k - \alpha_B^2(\lambda_A^2 + \lambda_I^2)}.$$

If $\lambda_A = 0$, $\theta^C = \dfrac{\alpha_B \lambda_I^2 (1 + b_S - c)}{4k - \alpha_B^2(\lambda_A^2 + \lambda_I^2)}$. Replacing for a_{NC}^P in $\theta^C(a)$, I obtain $\theta^* = \dfrac{\alpha_B \lambda_I^2 (1 + b_S - c)}{2k - \alpha_B^2(\lambda_A^2 + \lambda_I^2)}$. So $\theta^C < \theta^*$.

Appendix L1: proof of Proposition 6. The general expression of banks' profits is not modified. At stage 4 of the game, banks choose the transaction fees that maximise their profits. Solving for the first-order condition yields[23]:

$$\frac{\partial \pi_I}{\partial f} = (1 + \alpha_S \theta - m)(1 + \alpha_B \theta - a + c_I - 2f) = 0,$$

and

$$\frac{\partial \pi_I}{\partial f} = (1 + \alpha_B \theta - f)(1 + \alpha_S \theta + a + c_A - 2m) = 0.$$

The optimal transaction fees are

$$f^*(a, \theta) = \frac{1 + \alpha_B \theta - a + c_I}{2},$$

and

$$m^*(a, \theta) = \frac{1 + \alpha_S \theta + a + c_A}{2}.$$

The transaction volume at equilibrium of stage 4 is given by

$$\begin{aligned} V(a, \theta) &= \frac{1}{4}(1 + \alpha_B \theta + a - c_I)(1 + \alpha_S \theta - a - c_A) \\ &= M_A(a, \theta) M_I(a, \theta), \end{aligned}$$

and banks' profits are expressed as follows:

$$\pi_I = M_A(a, \theta)(M_I(a, \theta))^2 - C_I(\theta_I),$$

and

$$\pi_A = M_I(a, \theta)(M_A(a, \theta))^2 - C_A(\theta_A).$$

[23] The second-order condition is verified.

At stage 3 of the game, banks still choose their levels of quality such that the marginal benefits of investing in quality are equal to the marginal costs. However, the sign of the strategic effects is more difficult to determine, because the levels of quality are not necessarily strategic complements. The strategic complementarity of θ_I and θ_A depends now on the interchange fee.

I give a sketch of proof of this result for the Acquirer:

$$\frac{\partial^2 \pi_A}{\partial \theta_A \theta_I} = \frac{\partial V(a,\theta)}{\partial \theta_I} \times \frac{\partial \pi_A}{\partial \theta_A} + \frac{\partial V(a,\theta)}{\partial \theta_A} \frac{\partial M_A(a,\theta)}{\partial \theta_I} + \frac{\partial^2 M_A}{\partial \theta_I \theta_A} V(a,\theta) + \frac{\partial^2 V(a,\theta)}{\partial \theta_I \theta_A} M_A(a,\theta).$$

Since $\partial^2 V/\partial \theta_I \theta_A = (\alpha_B \alpha_S \lambda_I \lambda_A)/2$, and $\partial^2 M_A/\partial \theta_I \theta_A = 0$,

$$\frac{\partial^2 \pi_A}{\partial \theta_A \partial \theta_I} = \frac{\alpha_S \lambda_A \lambda_I}{4} (\alpha_S + \alpha_B + 2\alpha_S \alpha_B \theta + (\alpha_S - \alpha_B) a - \alpha_S c_I - \alpha_B c_A + \alpha_B + \alpha_B \alpha_S \theta - \alpha_B (a + c_A)).$$

Since $\dfrac{\partial R_A}{\partial \theta_I} = -(\dfrac{\partial^2 \pi_A}{\partial^2 \theta_A})^{-1} \dfrac{\partial^2 \pi_A}{\partial \theta_A \theta_I}$, the previous expression shows that the sign of $\dfrac{\partial R_A}{\partial \theta_I}$ depends on the interchange fee.

If the levels of quality are exogenous, as in Schmalensee (2002), the payment system chooses an interchange fee which balances demands between each side of the market, and which may leave some positive margin to the Acquirer. The profit of the payment system is expressed as follows:

$$\begin{aligned} \pi_S^E &= \pi_I + \pi_A \\ &= V(a,\theta)(M_I(a,\theta) + M_A(a,\theta)). \end{aligned}$$

Solving for the first-order condition yields[24]:

$$\frac{d\pi_S^E}{da} = \frac{1}{8}(2 + (\alpha_S + \alpha_B)\theta - c)((\alpha_S - \alpha_B)\theta - 2a + c_I - c_A) = 0.$$

The optimal interchange fee is $a^E(\theta) = \left[(c_I - \alpha_B \theta) - (c_A - \alpha_S \theta)\right]/2$.

Appendix L2: proof of Proposition 7. If the levels of quality are endogenous to the model, the general expression of the derivative of banks' profits with respect to the interchange fee is still

$$\frac{d\pi_S}{da} = \frac{\partial}{\partial a}\left(V(a,\theta) M_S(a,\theta)\right) + S_I(a) + S_A(a),$$

[24] The second order condition is verified.

where $\dfrac{\partial}{\partial a}\left(V(a,\theta)M_S(a,\theta)\right) = \dfrac{d\pi_S^E}{da}$. If the constraint $\pi_A \geq 0$ is not binding, the optimal interchange fee satisfies to the following equation:

$$a^P = a^E((\theta^*)(a^P)) + \frac{4}{2 + (\alpha_S + \alpha_B)(\theta^*)(a) - c}\left(S_I(a^P) + S_A(a^P)\right).$$

This equation shows that, if the strategic effects are not equal to zero, the optimal interchange fee is no longer set such that banks' marginal costs are equalized, because the marginal costs must be adjusted by the strategic effects:

$$c_I - a^P - \alpha_B\theta - \frac{2\left(S_I(a^P) + S_A(a^P)\right)}{2 + (\alpha_S + \alpha_B)(\theta^*)(a^P) - c} = c_A + a^P - \alpha_S\theta - \frac{2\left(S_I(a^P) + S_A(a^P)\right)}{2 + (\alpha_S + \alpha_B)(\theta^*)(a^P) - c}.$$