Licensing Standard-Essential Patents with Costly Enforcement*

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Abstract

We study the interaction between the holder of a standard-essential patent (SEP) and two downstream firms using the patented technology to design standard-compliant products. The SEP holder approaches the downstream firms simultaneously in the shadow of patent litigation and is subject to fair, reasonable, and non-discriminatory (FRAND) licensing requirements. We show that the patent holder faces a litigation credibility constraint and a license acceptability constraint when setting its licensing terms. For patents of intermediate strength, there is no royalty that allows the patent holder to reconcile these constraints. Consequently, it cannot license its technology and must go to court against infringers. We show that the availability of an injunction improves the patent holder's ability to license its technology, but it tends to inflate the royalty rate for implementers.

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1 Introduction

Technological standards play a major role in many industries, particularly in the Information and Communication Technology (ICT) sector, and often incorporate patent-protected technologies. Some of these patents are essential to the implementation of the standard, which means that a manufacturer developing standard-compliant products necessarily infringes the standard's essential patents.

To prevent hold-up by owners of standard-essential patents (SEPs), standard-setting organizations (SSOs) generally require them to offer licenses on Fair, Reasonable, and Non-discriminatory (FRAND) terms. Yet, we observe many litigation cases between SEP holders and implementers. For example, the smartphone patent wars have involved large companies, such as Apple, Google and Samsung, among others, in many lawsuits since 2010.[1]

One possible reason for the large number of litigation cases over SEPs is that FRAND commitments set *ex-ante* are too loose, leading to disputes between the different parties *ex-post*. Another explanation is that some standard implementers may be tempted to deliberately avoid licensing (i.e., "hold-out"), because, in the worst case, they will be sued and forced to pay the FRAND licensing fee.[2]

In this paper, we provide a theoretical framework to study the licensing and litigation strategy of a SEP owner, subject to FRAND commitments, in the shadow of patent litigation.

[1]See `https://en.wikipedia.org/wiki/Smartphone_patent_wars`. For statistics on the number of SEP litigation cases, see also European Commission (2014).

[2]See, e.g., Geradin (2010). In a 2017 Communication, the European Commission mentions these two sources of litigation over SEPs, writing that "the licensing and enforcement of SEPs is not seamless and may lead to conflicts. Technology users accuse SEP holders of charging excessive licensing fees based on weak patent portfolios and of using litigation threats. SEP holders claim that technology users 'free ride' on their innovations and consciously infringe intellectual property rights (IPR) without engaging in good faith licensing negotiations." (Communication from the Commission to the European Parliament, the Council and the European Economic and Social Committee, Setting out the EU approach to Standard Essential Patents, COM(2017)712 final, November 29, 2017.)

We develop a model in which the SEP holder seeks to sell fair, reasonable, and non-discriminatory (FRAND) licenses to two rival firms competing in the downstream market for standard-compliant products. A strict non-discrimination requirement implies that the patent holder must offer the same licensing terms to both firms.[3] We assume that the licensing terms are reasonable if the downstream firms can be active in the product market when they pay the license fee. The SEP holder approaches the downstream firms simultaneously and offers them a license to use its patented technology.[4] Each downstream firm can either accept the license or reject it, in which case it infringes the patented technology. The SEP holder can then decide to sue the firms that infringe its patent.

When the patent owner sues an infringer, both parties incur litigation costs. The litigation results in the patent being either upheld by the court with some probability or invalidated. If the patent is upheld, the court forces the downstream firms to pay the royalty rate the patent holder asked but does not award any further compensation. If the patent is invalidated, any previously signed license is canceled.

In setting its license fee, the SEP holder must consider the impact of the fee on the demand from potential licensees and its ability to defend its patent rights in court.

First, the patent holder should set a royalty rate low enough to make downstream firms willing to take a license, which we refer to as the *license acceptability constraint*. However, the firms' willingness to accept the license is constrained by a "hold-out" effect. In this paper, we define "hold-out" as the incentive of a downstream firm to reject the license offer, because, in the worst case, if the patent holder goes to court and the patent is upheld, it will have to pay the FRAND royalty rate, and in the best case, if the patent is invalidated, it will not have to pay any royalties.[5] Besides, if one firm rejects the license

[3]Non-discrimination requires that prospective licensees shall not be treated differently without an objective reason. This translates into strictly similar licensing terms in our main setting. We relax the assumption of strict non-discrimination in an extension of the model where the SEP holder is allowed to discriminate in a sequential licensing game.

[4]In Section 5, we show that we obtain the same results if the SEP holder approaches the potential licensees sequentially.

[5]Our definition is line, for instance, with Epstein and Noroozi (2017), who define "patent hold-out" as a situation where "an implementer refuses to negotiate in good faith with an innovator for a license to valid patent(s) that the implementer infringes, and instead forces the innovator to either undertake significant litigation costs and time delays to extract a licensing payment through a court order, or else

offer, it is even harder for the patent holder to persuade the other firm because it would be at a cost disadvantage in the downstream market.

Second, the patent holder must be credible in threatening the downstream firms with litigation if they infringe its patent. Litigation is a credible threat if the SEP holder sets a sufficiently high royalty rate, which we refer to as the *litigation credibility constraint.* However, the threat is weakened if a downstream firm has agreed to take a license. In this case, the patent holder faces an opportunity cost of going to court; if the patent is invalidated, it will lose licensing revenues from its licensee.

Therefore, the patent holder faces a trade-off when offering a license to a given firm. On the one hand, if the offer is attractive and the firm accepts it, the other firm is more willing to follow suit due to the absence of a cost disadvantage. On the other hand, the threat of litigation against the other firm is weaker because of the risk of losing royalty revenues from the first licensee. While the first effect is a source of strategic complementarity in licensing decisions, the second effect is a source of strategic substitution.

If the patent is weak, we show that litigation is not a credible threat; therefore, the patent holder cannot enforce its patent rights. Conversely, if the patent is strong, the patent holder's market power is unconstrained, and it can sell a license at the monopoly royalty rate to both downstream firms.

For patents of intermediate strength, the patent holder's market power is constrained by the license acceptability constraint and the litigation credibility constraint. We show that there is a range of relatively low patent strengths where the patent holder cannot reconcile the two constraints. The patent holder would have to set a high royalty rate to credibly threaten litigation against an infringer. However, firms are unwilling to take a license at high royalty rates. They prefer to hold out because there is a reasonable chance that the patent will be invalidated, and if it is upheld, they will have to pay no more than the FRAND rate. In this case, at best, the patent holder can license its technology to one firm at a low royalty rate but not to both. At worst, it cannot license its technology to any of them and must go to court against the infringers. The latter outcome is inefficient

to simply drop the matter because the licensing game is no longer worth the candle."

for the patent holder but also for the downstream firms, because the patent holder then sets the monopoly royalty rate in anticipation of litigation.

One may wonder whether the ND (non-discriminatory) component of FRAND is responsible for the patent holder's inability to license its technology. We thus allow the patent holder to set discriminatory royalties and show that our qualitative results remain valid. For any level of patent strength, the number of licenses that the patent holder can sell is the same, whether the patent holder charges discriminatory royalties or a uniform royalty rate.

To improve the patent holder's ability to license its technology, we consider the possibility of an injunction order issued by the court, allowing the patent holder to renegotiate the royalty rate in the shadow of an injunction threat. The availability of an injunction relaxes the patent holder's litigation credibility and license acceptability constraints. Consequently, it allows the patent holder to sell a higher number of licenses for a patent of intermediate strength than in the baseline case. However, it also allows the patent holder to charge a higher royalty rate to the detriment of downstream firms.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, we set up the model. We solve for the equilibrium and present our main findings in Section 4. In Section 5, we consider sequential licensing decisions, cost asymmetries between downstream firms, and discriminatory royalties. In Section 6, we show that if the court issues an injunction order when the patent is upheld, the patent holder's ability to license its technologies is improved. We conclude in Section 7.

2 Related literature

Our paper contributes to several strands of literature. First, it contributes to the literature on licensing of standard-essential patents (SEPs) and FRAND commitments. This literature focuses on the hold-up problem, which arises when, after standardization has taken place, there is no alternative to SEPs for implementers, allowing SEP holders to claim abusive royalties *ex-post*. Choi (2016) discusses how the court procedure can solve the hold-up problem. Other authors argue that FRAND commitments cannot properly solve this problem and propose alternative mechanisms, such as *ex-ante* bilateral negotiations between SEP owners and implementers (Layne-Farrar et al., 2009), option-to-license contracts (Ganglmair et al., 2012) or mandatory *ex-ante* price caps on royalties (Lerner and Tirole, 2015). Our focus is different. We abstract from the hold-up problem and study the licensing strategy of a SEP owner facing multiple downstream firms in the shadow of patent litigation. We show that litigation credibility and the hold-out problem significantly constrain the patent holder's ability to license its technology.

Miao (2016) also studies the optimal licensing strategy of a SEP owner under different ownership structures. He shows that when the patent holder is only active upstream, as in our framework, a revenue-sharing royalty maximizes licensing revenues by replicating the integrated monopoly outcome. The key difference with our paper is that his setup ignores the probabilistic nature of SEPs and the possibility of infringement and litigation, which play a critical role in our analysis.

Most of the literature on FRAND licensing focuses on the "fair and reasonable" part of the commitment and ignores the "non-discriminatory" (ND) part. Two exceptions are Layne-Farrar et al. (2009) and Li and Shuai (2019). Layne-Farrar et al. (2009) argue that the ND component is critical to ensuring reasonable royalties, especially for licensees contracting *ex-post*. Li and Shuai (2019) show that the ND commitment alleviates the hold-up problem and leads to higher investments by implementers. In our framework, the ND component plays a marginal role; allowing the SEP holder to set discriminatory royalties only marginally improves its ability to license its technology.

Our paper also relates to the literature on licensing of probabilistic patents,[6] which studies the licensing of a cost-reducing innovation in the shadow of patent litigation (see, e.g., Farrell and Shapiro (2008), Encaoua and Lefouili (2009) and Amir et al. (2014)).[7] Our contribution to this literature is twofold. First, we study the licensing strategy for a standard-essential patent, which allows designing standard-compliant products but

[6]An earlier literature focused on licensing strategies for ironclad patents. See, e.g., Kamien and Tauman (1984) and Katz and Shapiro (1985, 1986).

[7]Other relevant contributions in this stream of literature, but with a different focus, are Aoki and Hu (1999) and Choi (2010).

brings no cost reduction and has no alternative for the potential licensees. While in the licensing literature, a non-licensee has access to an inferior (backstop) technology, in our framework, the worst-case scenario for an infringing producer is to be forced to pay the FRAND rate if the patent is upheld in court. Hence, hold-out can occur. Second, we explicitly model the patent holder's litigation decision. In contrast, Farrell and Shapiro (2008) assume that it always litigates infringers,[8] and Encaoua and Lefouili (2009) and Amir et al. (2014) consider that producers initiate the litigation. We show that the patent holder faces a litigation credibility constraint, which critically limits its ability to license its technology.

Our paper is also related to the literature on sequential patent litigation. The closest papers to ours are Choi (1998) and Choi and Gerlach (2018).[9] Choi (1998) studies the litigation strategy of a vertically-integrated patent holder facing two potential entrants that threaten to imitate its technology. If the patent holder sues an infringing entrant, the validity of its patent is revealed to further entrants. Choi shows that with a patent of intermediate strength, the patent holder accommodates the first entrant to maintain uncertainty about the patent's validity for the second entrant.[10] As an extension of our baseline model with simultaneous licensing decisions, we develop a sequential litigation setting similar to Choi (1998)'s but allow the patent holder to set the licensing terms for its technology. We then study how the litigation credibility constraint affects the patent holder's decision of those terms. Different from Choi (1998), we also study a non-practicing (pure-upstream) patent holder.

Choi and Gerlach (2018) study the sequential negotiations between a non-practicing entity (NPE) and two potentially infringing firms. Litigating the first firm generates an informational externality, as the outcome of the litigation informs of the likelihood that

[8]Farrell and Shapiro (2008) state that modeling the litigation decision of the patent holder would be a natural and interesting extension of their framework.

[9]Other relevant papers include Choi and Gerlach (2009) and Palikot and Pietola (2019). Choi and Gerlach (2009) show that the formation of a patent pool allows independent patent holders to charge higher royalties to a downstream firm by strengthening their patent portfolio and allowing them to internalize litigation externalities. Palikot and Pietola (2019) study the impact of "pay-for-delay" deals on the litigation strategy of a patent holder. They show that these deals generate a payoff externality between entrants and allow the patent holder to adopt a divide-and-conquer strategy.

[10]See also Duchêne and Serfes (2012), who extends Choi (1998)'s framework by introducing asymmetric information and the possibility of settlement.

the second firm will be found to be infringing. Choi and Gerlach show that when this information externality is positive, the NPE can extract higher revenues by approaching the firms sequentially rather than in isolation. There are two key differences between Choi and Gerlach (2018) and our paper. First, Choi and Gerlach do not study the patent holder's choice of licensing terms and whether downstream firms should accept or reject the license offer. Therefore, there is no license acceptability constraint affecting the patent holder's decision. Second, in Choi and Gerlach (2018)'s setting, the patent holder does not face an opportunity cost when negotiating with the firms. This is because the litigation is about infringement, not the validity of the patent, and the outcome of the negotiation with the first firm is sunk when turning to the second firm. We find that this opportunity cost severely limits the patent holder's litigation credibility.

Finally, our paper also relates to the law and economics literature on settlements.[11] This literature highlights the role of informational and payoff externalities in the negotiations between a defendant and multiple plaintiffs. Che and Yi (1993) show that if a defendant is unlikely to win a trial against a first plaintiff, he should settle to avoid setting an unfavorable precedent (a negative informational externality). Daughety and Reinganum (2002) consider two informational externalities in the interaction between a defendant and a first plaintiff; a "publicity effect," whereby the first plaintiff's actions may inform a second plaintiff of the connection between her problem and the defendant, and a "learning effect," whereby the defendant's decision in the first case may signal the strength of his case to the second plaintiff. Daughety and Reinganum show that "strong" defendants, with a low probability of being found liable, go to court against the first plaintiff, "weak" defendants settle with both plaintiffs and, finally, defendants of intermediate strength settle with the first plaintiff but go to trial against the second one.

Che and Spier (2008) highlight the role of payoff externalities in settlement agreements. In their setup, litigation costs are shared among the plaintiffs who go to court. Thus, when a plaintiff decides to settle with the defendant instead of going to court, she imposes a negative payoff externality on the other plaintiffs. Che and Spier show that the

[11] See Spier (2007) for a comprehensive review of this literature.

defendant can then adopt a divide-and-conquer strategy, discriminating among otherwise identical plaintiffs by making more favorable offers to some than to others.

As in this literature, we study the role of informational and payoff externalities, but in a different context, where a SEP owner offers a license to implementers in the shadow of patent litigation. When we consider sequential licensing decisions, an informational externality arises because by going to court, the SEP owner reveals the true validity of its patent. There is also a payoff externality, since the decision of the first producer to take or refuse the license affects competition in the downstream market. What is specific to our setup compared to the literature on settlements is that the magnitude of the payoff externality depends on the SEP owner's choice of licensing terms.

3 The model

We study the interaction between the upstream holder of a standard-essential patent (SEP) and two downstream firms that use its patented technology to design standard-compliant products. Due to FRAND rules, the patent holder cannot discriminate between the two downstream firms and thus sets a uniform per-unit royalty fee for access to its technology. We model the interaction between the patent holder and the downstream firms for a licensing agreement in the shadow of patent litigation.

Firms. The patent holder, firm P, owns a patent on a technology that is an essential input for two downstream firms, 1 and 2.[12] Firm P is not active in the downstream market and approaches the downstream firms simultaneously, offering them a uniform per-unit royalty rate $r \geq 0$ for using its patented technology.

The two downstream firms are identical *ex-ante*.[13] *Ex-post*, they may differ if one accepts the license, but not the other. The patent holder makes take-it-or-leave-it licensing offers, and each downstream firm can either accept or reject the offer. Refusing to pay for

[12]This means that the downstream firms can be active if, and only if, they use the patented technology. There is no backstop technology that they could use.

[13]The model can be easily be extended to $n \geq 2$ downstream firms by assuming that the first $n-1$ downstream firms make the same licensing decision (i.e., to accept or reject the license offer).

the license exposes a downstream firm to litigation from the patent holder. However, as we will describe below, it does not otherwise preclude from using the patented technology. Finally, we assume that all firms are risk-neutral.

Product market. The two downstream firms compete in the product market, but we do not specify the type of competition. For the sake of simplicity, we assume that the downstream firms have the same constant marginal cost of production, which we normalize to zero. Therefore, if it has to pay royalties to the patent holder, a downstream firm has a perceived marginal cost of r, whereas its perceived marginal cost is 0 if it does not pay for the license.

Let c_i denote the perceived marginal cost of firm $i = 1, 2$, and l_i its licensing decision, where $l_i = 1$ if firm i accepts the license, and $l_i = 0$ if it rejects it. With these notations, we have $c_i = l_i r$.

We assume that there is a unique equilibrium to the competition game, for any couple of marginal costs $(c_1, c_2) \in \mathbb{R}_+^2$. We denote by $q_i(c_1, c_2)$ the equilibrium quantity of firm $i = 1, 2$, and by $p_i(c_1, c_2)$ its equilibrium price. We assume that the equilibrium is symmetric, in the sense that we have $q_1(c_1, c_2) = q_2(c_2, c_1)$ and $p_1(c_1, c_2) = p_2(c_2, c_1)$.

Firm i's equilibrium profit, gross of any litigation costs, is then given by

$$\pi_i(c_1, c_2) = (p_i(c_1, c_2) - c_i)\, q_i\,(c_1, c_2)\,,$$

and firm P's profit, again gross of litigation costs, is

$$\Pi_P(l_1, l_2; r) = (l_1 q_1(l_1 r, l_2 r) + l_2 q_2(l_1 r, l_2 r))\, r.$$

If the firms have the same perceived marginal costs, we will drop the subscripts; i.e., $p_1(c, c) = p_2(c, c) = p(c, c)$, $q_1(c, c) = q_2(c, c) = q(c, c)$, and $\pi_1(c, c) = \pi_2(c, c) = \pi(c, c)$.

We make the following (standard) assumptions on equilibrium quantities and profits:

A1. $q_2(0, r) \leq q(r, r) \leq q(0, 0)$;

A2. $\pi_2(0, r) \leq \pi(r, r) \leq \pi(0, 0)$;

A3. $\pi_1(0, r) \geq \pi(r, r)$.

Assumptions A1 and A2 mean that a downstream firm has a lower quantity and makes lower profits when it competes at a cost disadvantage with its rival, which is the case if the former firm pays royalties but the latter does not. These two assumptions also imply that the downstream firms have larger quantities and make higher profits if they do not pay royalties than if they both do. Finally, Assumption A3 means that a firm makes higher profits if it does not pay royalties while its rival does than if they both do.

Our main analysis relies on this general model of competition, but in the paper, we will also use as an illustrative example a model of Cournot competition with homogeneous products with the linear inverse demand $P(Q) = 1 - Q$. This specific model of competition satisfies our assumptions on the existence and symmetry of the equilibrium of the product market as well as Assumptions A1-A3.

Litigation. If a downstream firm or both of them refuse the license, the patent holder may decide to litigate the infringer(s). Litigation is costly for all parties, each of which incur a fixed cost $L > 0$ independent of the outcome.

Firm P's patent has a probability $\theta \in (0, 1)$ of being validated during litigation.[14] Validation encompasses both the question of whether the patent office should have granted the patent and whether the patent is pertinent to the standard. The strength θ of the patent is common knowledge to all firms.

If the patent is held valid by the court, we assume that the downstream firms respect the patent and pay the license fee.

Conversely, if the patent is held invalid, we assume that the downstream firms are free to produce without paying the license fee. This corresponds to the "collateral estoppel" doctrine,[15] according to which a patent owner cannot assert its intellectual property

[14]In line with the literature on probabilistic patents, we assume that the patent is not ironclad, i.e., $\theta < 1$. This assumption captures the imperfect nature of the patent granting process, where patents that should not have been granted end up being invalidated in court. We assume that *ex-ante* the parties cannot foresee with certainty the outcome of a court revision of the patent. For a more complete discussion, see Farrell and Shapiro (2008). For empirical evidence on litigation of standard-essential patents, see, e.g., European Commission (2014).

[15]See, e.g., Spier (2007) for a discussion of the collateral estoppel doctrine.

rights if its patent has been invalidated. We assume that this is the case even if one of the downstream firms had previously agreed to pay for the license.[16]

FRAND licensing. In our framework, the holder of the essential patent has to comply to a FRAND (fair, reasonable, and non-discriminatory) licensing rule. First, as already explained, this means that firm P cannot discriminate between the two downstream firms with respect to its licensing terms; in other words, the royalty rate r must be uniform.[17] Second, the license offer must be reasonable. We interpret this rule loosely as requiring that the licensees must be active in the product market.[18] Formally, P must set a royalty rate r low enough so that $q(r, r) > 0$ when both firms are licensed and $q_2(0, r) > 0$ when only one is.[19]

For further comparison, we define the monopoly royalty rate r^m that the patent holder would set if its patent were ironclad (i.e., $\theta = 1$) and no litigation could take place (e.g., because litigation is prohibitively costly). Assume that the monopoly royalty revenues $2rq(r, r)$ are quasi-concave in r; the monopoly royalty rate is then given by:

$$r^m \equiv \arg\max_r 2rq(r, r).$$

Note that in our framework with a probabilistic patent, the patent holder will never set a licensing fee above r^m in equilibrium.

Timing. The game unfolds in three stages, as follows:

1. The patent holder, firm P, sets the per-unit royalty rate, r, and offers a license to firm 1 and firm 2.

[16]Farrell and Shapiro (2008) make the same assumption. Industry experts confirmed to us that this is how it would work in practice.

[17]In practice, this uniform pricing obligation applies when the licensees are "similarly situated" (see, e.g., Carlton and Shampine (2013)). This is the case here, as the downstream firms compete in the same product market and are *ex-ante* identical.

[18]Therefore, as Lerner and Tirole (2015), we adopt a "pessimistic view of FRAND," which does not constrain the patent holder's *ex-post* market power. Note that our restriction on the level of royalty would be just a minimum requirement. Although Choi (2016) argues that the reasonable component of FRAND is difficult to define, the law and economics literature on FRAND suggests that the "reasonable" royalty rate must be based on an *ex-ante* benchmark (see, e.g., Swanson and Baumol (2005)).

[19]Note that our symmetry assumptions imply that $q_1(r, 0) = q_2(0, r)$. So, it does not matter here which firm is the only licensee.

2. The downstream firms decide simultaneously and non-cooperatively whether to accept the license.

3. Firm P can litigate the firms that have refused the license.

4. Profits are realized.

We look for the Subgame Perfect Equilibrium of this game.

We assume that firm P commits to a uniform royalty rate r. There is no possibility of renegotiation for the licensees, and the patent holder cannot change its royalty rate after a firm has refused the license offer.

4 The equilibrium

In this section, we solve for the equilibrium of the game. As standard, we proceed by backward induction and start with the last stage.

4.1 Patent holder's litigation decision

At Stage 3, firm P can litigate the infringing firms. If no firm has accepted the license, firm P can go to court and try to force both firms to pay the license fee, which happens if the patent is validated in court (i.e., with probability θ). Alternatively, it can renounce its royalties altogether and save the litigation cost. Therefore, firm P goes to court against *both* firms if the following litigation condition holds:

$$2\theta rq(r,r) \geq L. \tag{LC_B}$$

If one firm has accepted the license but not the other, firm P faces a different trade-off. If it sues the infringing firm, the patent's validity is confirmed or denied in court, as in the previous case. If it does not go to court, firm P saves the litigation cost, but it now also derives royalty revenues from a (single) licensee. Thus, firm P decides to go to court against a *single* infringer if the following litigation condition holds:[20]

$$2\theta r q(r,r) \geq L + r q_1(r,0). \qquad (LC_S)$$

Compared to the litigation condition (LC_B), condition (LC_S) incorporates an opportunity cost for the patent holder, which is equal to $rq_1(r,0)$. Firm P has signed a licensing contract with one firm, and it can earn royalty revenues from this firm. By going to court against the other infringing firm, firm P runs the risk of seeing its patent invalidated and, hence, losing these royalty revenues. The patent holder's threat to litigate an infringer is thus less credible if it has signed a licensing contract with one downstream firm.

We can express conditions (LC_B) and (LC_S) as conditions on the strength θ of the patent. For $r > 0$, we define the following threshold values of θ:

$$\underline{\theta}(r) \equiv \frac{L}{2rq(r,r)} \quad \text{and} \quad \overline{\theta}(r) \equiv \frac{L + rq_1(r,0)}{2rq(r,r)} > \underline{\theta}(r).$$

Condition (LC_B) holds if and only if $\theta \geq \underline{\theta}(r)$, whereas condition (LC_S) holds if and only if $\theta \geq \overline{\theta}(r)$, with $\overline{\theta}(r) > \underline{\theta}(r)$.

The following lemma characterizes firm P's optimal litigation decision at Stage 3.

Lemma 1. *Firm P's optimal litigation decision is as follows:*

(i) If both firms have refused the license, P goes to court if (LC_B) holds, i.e., $\theta \geq \underline{\theta}(r)$;

(ii) If only one firm has refused the license, P goes to court if (LC_S) holds, i.e., $\theta \geq \overline{\theta}(r)$.

4.2 Downstream firms' licensing decisions

At Stage 2, the two downstream firms decide simultaneously and non-cooperatively whether to accept the license, anticipating firm P's litigation decision at the next stage.

A firm's decision to take a license depends on its rival's decision and how credible is the patent holder's litigation threat.

[20] Note that due to our symmetry assumptions, we have $q_1(r,0) = q_2(0,r)$. Therefore, condition (LC_S) is the same whether firm 1 or firm 2 is the single licensee.

First, consider the case where neither (LC_B) nor (LC_S) holds. Since litigation is not a credible threat in any subgame, each firm can safely refuse to pay for the license to obtain larger profits. Hence, it is a dominant strategy for each firm to refuse the license.

Now, consider the case where (LC_B) holds, but not (LC_S). Firm P can credibly threaten to go to court if no firm accepts the license. However, if one firm accepts it, it cannot credibly threaten to litigate the other firm if it infringes its patent. Thus, firm P can bring at most one firm to take a license.

Consider then firm i's decision to accept the license, given the decision taken by the other firm $j \neq i$. If firm j accepts the license, there is no risk for firm i of being brought to court when refusing the license. So, its best response is to refuse it.

If, instead, firm j refuses the license, firm i has to compare its expected profits if it accepts the license and if it refuses it and firm P goes to court. If firm i accepts the license and pays royalties, it will compete at a cost disadvantage against firm j, which will not be sued for infringing the patent. Firm i's profit in this case is $\pi_1(r,0)$.[21] Conversely, if firm i refuses the license, firm P will go to court against both firms. The patent will then either be validated (with probability θ) or invalidated (with probability $1-\theta$) for all firms. Firm i's expected profit in this case is $\theta\pi(r,r) + (1-\theta)\pi(0,0) - L$. Therefore, firm i accepts the license if and only if $\pi_1(r,0) \geq \theta\pi(r,r) + (1-\theta)\pi(0,0) - L$, that is, if the following participation constraint holds:

$$\pi(r,r) \geq \pi(0,0) - \frac{L}{1-\theta} + \frac{\pi(r,r) - \pi_1(r,0)}{1-\theta}. \qquad (PC_S)$$

Finally, consider the case where both (LC_B) and (LC_S) hold. In this case, firm P's litigation threat is credible whether only one firm infringes the patent or both do.

Consider first that firm j accepts the license. If firm i follows suit, both firms will pay royalties and obtain the same profit, $\pi(r,r)$. If firm i refuses the license, firm P will go to court, and the patent will be either validated (with probability θ) or invalidated (with probability $1-\theta$). Therefore, firm i accepts the license if and only if $\pi(r,r) \geq$

[21] Note our symmetry assumptions imply that $\pi_1(r,0) = \pi_2(0,r)$. Hence, it does not matter whether firm i is firm 1 or firm 2.

$\theta\pi(r,r) + (1-\theta)\pi(0,0) - L$, that is, if the following participation constraint holds:

$$\pi(r,r) \geq \pi(0,0) - \frac{L}{1-\theta}. \qquad (PC_B)$$

If firm j refuses the license, firm i's best response is to accept it. Indeed, firm P will go to court for sure since one firm at least refuses the license. Therefore, firm i is better off accepting the license to save on litigation costs. Litigation has a public good nature, and firm i free rides on its rival's decision to bring the patent holder to court by refusing the license offer.

Note that the worst possible outcome for a firm, if it infringes the patent, is having to pay the FRAND royalty rate, r. Therefore, firm P faces a potential "hold-out" problem; a firm may prefer risking litigation than paying for the license. Thus, firm P must offer sufficiently attractive licensing terms to make firms accept the license, as shown in conditions (PC_S) and (PC_B).[22]

Comparing the licensing conditions (PC_S) and (PC_B) further shows that, since $\pi(r,r) \geq \pi_1(r,0)$ from our assumptions, a firm is less willing to take a license if its rival does not take one. This is because, if the rival infringes the patent and is not sued, the firm accepting the license will compete at a cost disadvantage in the downstream market.

Therefore, firm P may face a trade-off when it offers a license to a firm. On the one hand, if the offer is sufficiently attractive that the firm accepts it, the other firm is more willing to follow suit due to the absence of a cost disadvantage. On the other hand, the litigation threat against the second potential licensee is weaker because of firm P's opportunity cost of going to court (i.e., the risk of losing royalty revenues from the first licensee). The former effect is a source of strategic complementarity in licensing decisions, whereas the latter is a source of strategic substitution.

Based on this analysis, we can characterize the downstream firms' best responses at the licensing stage, using the fact that condition (LC_B) holds if and only if $\theta \geq \underline{\theta}(r)$, and

[22] This hold-out effect does not arise in Farrell and Shapiro (2008) or Amir et al. (2014), because, in their settings, licensing concerns a non-essential technology and is not subject to FRAND requirements. If the patent is upheld in court, the infringing firm reverts its backstop technology, with a marginal cost v. Since $r \leq v$ necessarily (otherwise, firms would never buy a license), going to court is more costly for a downstream firm than when the patent holder is committed to FRAND terms.

condition (LC_S) holds if and only if $\theta \geq \overline{\theta}(r)$, with $\overline{\theta}(r) > \underline{\theta}(r)$.

Lemma 2. *Given the licensing decision of firm $j \neq i$, firm i's best response is as follows:*

(i) If $\theta < \underline{\theta}(r)$, it is a dominant strategy for firm i to refuse the license;

(ii) If $\underline{\theta}(r) \leq \theta < \overline{\theta}(r)$, firm i accepts the license if, and only if, firm j refuses it and (PC_S) holds;

(iii) If $\theta \geq \overline{\theta}(r)$, firm i accepts the license if firm j refuses it or if firm j accepts it and (PC_B) holds.

Proof. See Appendix A. □

As an illustration, Figure 1 below shows the best responses for the linear Cournot model, as a function of the patent strength θ (on the horizontal axis) and the royalty rate r (on the vertical axis).[23] In the figure, A stands for *accept* and R stands for *refuse* (the license offer).

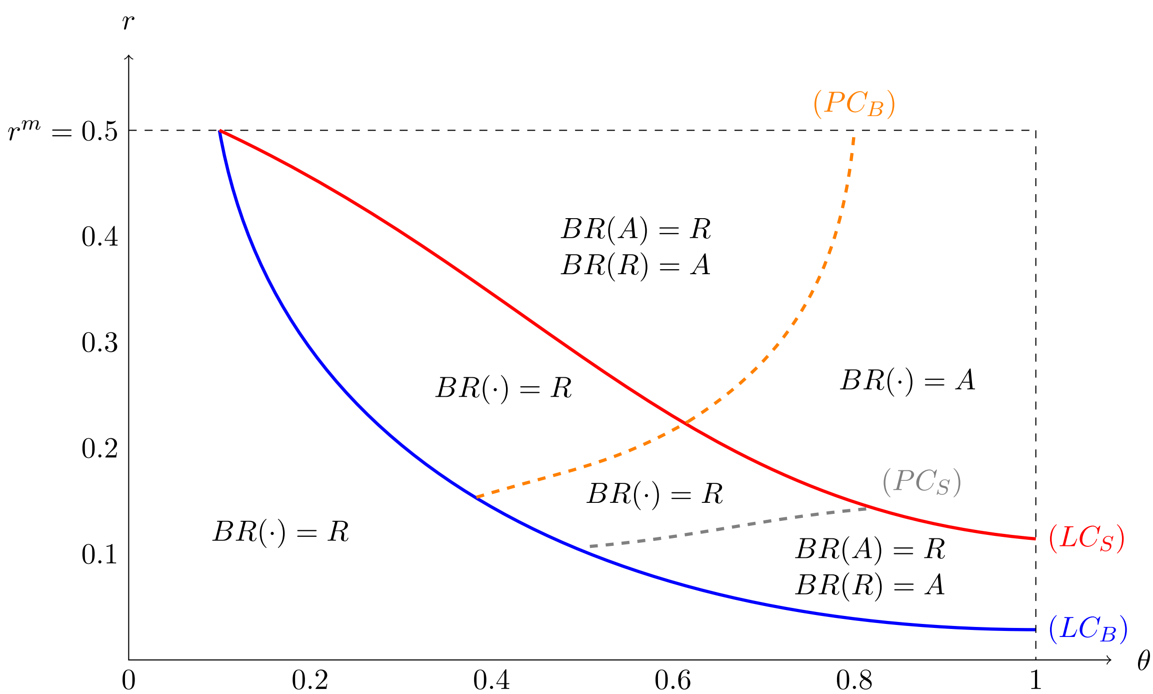

Figure 1: Best responses in the licensing subgame as a function of θ and r.[24]

[23]The analysis for the linear Cournot model is provided in Appendix B.

[24]This figure and the other figures in the paper were drawn for a litigation cost set to $L = 0.02$. Other values of the litigation cost lead to similar results.

Using the firms' best responses, we can characterize the equilibria of the simultaneous licensing game for a given royalty r.

Proposition 1. *For a given royalty rate r, the equilibrium outcome is as follows:*

(a) If $\theta < \underline{\theta}(r)$, both firms refuse the license and there is no litigation.

(b) If $\underline{\theta}(r) \leq \theta < \overline{\theta}(r)$, and (PC_S) holds, one firm accepts the license, the other firm refuses it, and there is no litigation. Otherwise, if (PC_S) does not hold, both firms refuse the license and firm P goes to court.

(c) If $\theta \geq \overline{\theta}(r)$, both firms accept the license if (PC_B) holds. Otherwise, if (PC_B) does not hold, one firm accepts the license, the other refuses it, and firm P goes to court.

Proposition 1 shows that the patent holder faces two constraints when setting its licensing terms, which affect the equilibrium outcome. First, it faces a litigation credibility constraint; it must charge a sufficiently high royalty rate to have a credible litigation threat, in particular when the opportunity cost effect is at play. Second, the patent holder faces a license acceptability constraint. Downstream firms may prefer going to court than accepting the license since, in the worst case, they will have to pay the FRAND royalty rate. The patent holder must set a sufficiently low royalty rate to mitigate this hold-out effect and bring downstream firms to accept the license offer.

Proposition 1 characterizes the equilibrium at Stage 2 for the general model. As an illustration, Figure 2 shows the equilibrium licensing and litigation decisions for the linear Cournot model as a function of the patent strength θ (on the horizontal axis) and the royalty rate r (on the vertical axis).

The area below condition (LC_B), represented by the blue solid line on the figure, corresponds to Case (a) in Proposition 1. Since firm P's litigation threat is not credible, the downstream firms refuse the license, and there is no litigation.

The area between the blue and red solid lines corresponds to Case (b) of the proposition. In this area, the downstream firms' licensing decisions tend to be strategic substitutes due to the opportunity cost of going to court with a single licensee. Thus, firm P can bring only one firm to take the license. Therefore, either firm P licenses only one

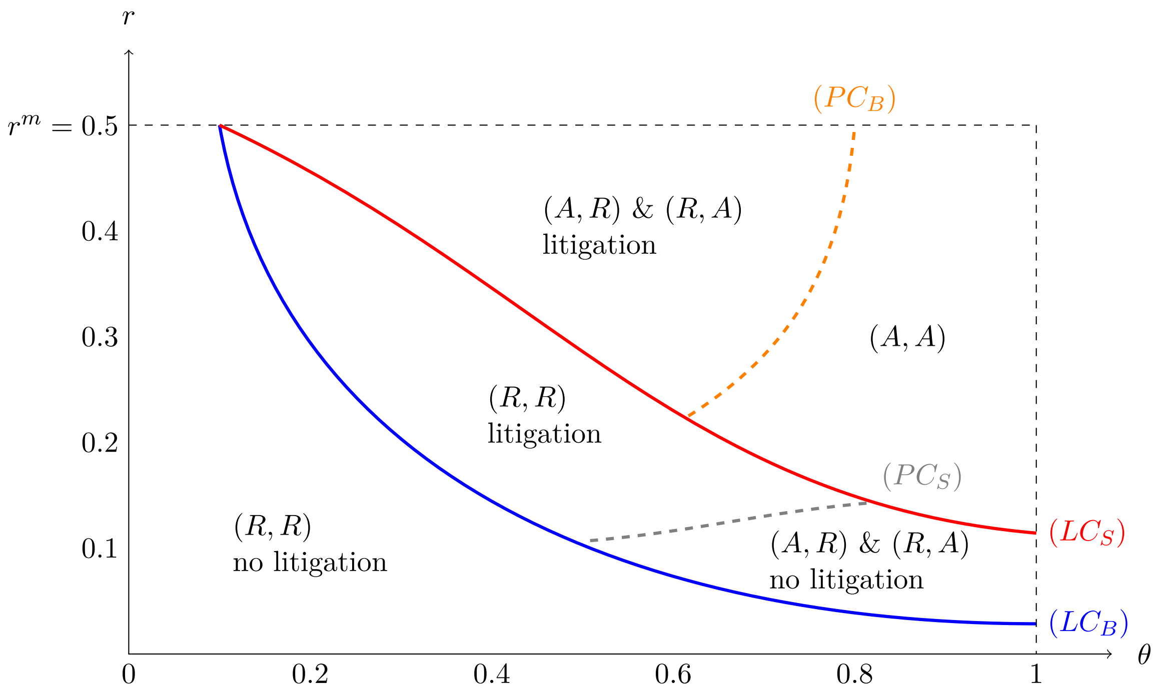

Figure 2: Equilibria of the simultaneous licensing game as a function of θ and r.

firm (if condition (PC_S) holds, i.e., we are below the gray line) or both firms refuse the license and firm P goes to court (if condition (PC_S) does not hold, i.e., we are above the gray line).

Finally, the area above the red solid line corresponds to Case (c) of the proposition. If condition (PC_B) holds (i.e., we are below the dotted orange line), the downstream firms' licensing decisions are strategic complements due to the absence of a cost disadvantage when taking a license. In this case, both firms accept the license in equilibrium. Otherwise, one firm accepts the license, the other refuses it, and firm P sues the infringing firm.

In Appendix C, we show that the representation of the equilibrium of the licensing game for the Cournot model in Figure 2 also applies to the general model under some conditions. More precisely, we show that there exists thresholds $r_L^B(\theta)$ and $r_L^S(\theta)$, both decreasing in θ, such that condition (LC_B) holds if and only if $r \geq r_L^B(\theta)$ and condition (LC_S) holds if and only if $r \geq r_L^S(\theta)$, with $r_L^S(\theta) \geq r_L^B(\theta)$, and thresholds $r_P^S(\theta)$ and $r_P^B(\theta)$, both increasing in θ, such that condition (PC_S) holds if and only if $r \leq r_P^S(\theta)$ and condition (PC_B) holds if and only if $r \leq r_P^B(\theta)$, with $r_P^B(\theta) \geq r_P^S(\theta)$, if the following conditions hold: (i) $\pi(r,r)$ is decreasing, (ii) $\partial\pi(r,r)/\partial r \geq \partial\pi_1(r,0)/\partial r$, and (iii) $-q'(r,r)r/q(r,r) \geq -q_1'(r,0)r/q_1(r,0)$. Condition (ii) means that a licensee's profits decrease faster with the royalty rate if only one firm is licensed than if they both are. Condition (iii) states that the demand with a single licensee is more elastic to the royalty rate than the demand with two licensees. In the rest of the paper, we assume that these conditions hold.

4.3 The patent holder's decision

In the first stage, the patent holder chooses its uniform royalty rate r to maximize its profit, anticipating how the downstream firms will react to its licensing offer in the following stage.

Proposition 1 shows that for a given patent strength θ, as the royalty rate r increases from zero to the monopoly royalty rate r^m, the equilibrium of the licensing game moves through various cases, with different licensing and litigation outcomes.

However, there are two special cases where the patent holder's choice of royalty rate is straightforward. First, the patent may be so weak that firm P cannot enforce its intellectual property rights for any royalty rate, and thus always makes zero profits. This happens when $\theta \leq \hat{\theta}_W$, where $\hat{\theta}_W \equiv \underline{\theta}(r^m)$. On Figure 2, the threshold $\hat{\theta}_W$ corresponds to the intersection of condition (LC_B) with the horizontal line $r = r^m$.

Conversely, the patent may be so strong that firm P can enforce its patent rights when it sets the monopoly royalty rate r^m. This happens when $\theta \geq \hat{\theta}_S$, where $\hat{\theta}_S \equiv 1 - L/(\pi(0,0) - \pi(r^m, r^m))$. On Figure 2, the threshold $\hat{\theta}_S$ corresponds to the intersection of condition (PC_B) with the horizontal line $r = r^m$. If $\theta \geq \hat{\theta}_S$, firm P thus sets the monopoly royalty rate, the two downstream firms accept the license and there is no litigation.

For intermediate levels of patent strength, between $\hat{\theta}_W$ and $\hat{\theta}_S$, the patent holder faces a trade-off between setting a high royalty rate to derive high per-unit royalty revenues and make its litigation threat more credible, and setting a low royalty rate to increase firms' willingness to accept the license. We assume that this case exists, that is, $\hat{\theta}_W < \hat{\theta}_S$.[25] We then define the following thresholds for the patent strength in this range: $\hat{\theta}_1$ corresponds to the intersection of conditions (LC_B) and (PC_S), while $\hat{\theta}_2$ corresponds to the intersection of conditions (LC_S) and (PC_B). We have $\hat{\theta}_1 \geq \hat{\theta}_W$ and $\hat{\theta}_2 \leq \hat{\theta}_S$ by construction. As we will see, firm P can sell a license to one firm only if $\theta \geq \hat{\theta}_1$, and to two firms only if $\theta \geq \hat{\theta}_2$.

Note that for this range of patent strength, between $\hat{\theta}_W$ and $\hat{\theta}_S$, firm P may trade off between (i) selling a license to one or both firms at a level of royalty acceptable by them, and (ii) going to court by setting a royalty rate that will not be accepted by the firms. In the latter case, the patent holder is better off setting the monopoly royalty rate r^m.

We can now characterize the equilibrium of the whole game.

Proposition 2. *The equilibrium outcome is as follows:*

(i) If $\theta < \hat{\theta}_W$, no firm is licensed and there is no litigation;

(ii) If $\theta \in [\hat{\theta}_W, \min\{\hat{\theta}_1, \hat{\theta}_2\})$, no firm is licensed and firm P goes to court;

(iii) If $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$, either only one firm is licensed at the royalty rate $r^\star$ such that (PC_S) binds, or no firm is licensed and firm P goes to court;

(iv) If $\theta \in [\hat{\theta}_2, \hat{\theta}_S)$, either the two firms are licensed at the royalty rate $r^\star$ such that (PC_B) binds, or no firm is licensed and firm P goes to court;

(v) If $\theta \geq \hat{\theta}_S$, the two firms are licensed at the monopoly royalty rate $r^\star = r^m$.

Proof. See Appendix D. □

If its patent is weak ($\theta < \hat{\theta}_W$), firm P is unable to enforce its patent rights, and therefore, no firm takes a license. Conversely, if its patent is strong ($\theta \geq \hat{\theta}_S$), firm P's market power is unconstrained and it can license the two firms at the monopoly royalty rate.

[25] In the Cournot model, this is the case if the litigation cost, L, is low enough, that is, $L < 1/18$.

For a patent of intermediate strength ($\theta \in [\hat{\theta}_W, \hat{\theta}_S)$), however, the patent holder's market power is constrained by the litigation credibility and license acceptability constraints.

If the patent is strong enough ($\theta \in [\hat{\theta}_2, \hat{\theta}_S)$), the binding constraint is the license acceptability constraint (condition (PC_B)). Firm P can sell a license to both firms by setting a royalty rate low enough so that (PC_B) binds. Alternatively, firm P can decide to go to court, in which case it sets the monopoly royalty rate.

For a patent of lower strength ($\theta \in [\hat{\theta}_W, \hat{\theta}_2)$), the litigation credibility constraint kicks in. On the one hand, the patent holder's royalty rate must be low enough to make the downstream firms willing to take a license (i.e., (PC_B) should hold). On the other hand, it must be high enough to ensure the patent holder's litigation credibility (i.e., (LC_S) should hold). For this range of patent strength, however, there is no royalty rate that satisfies both constraints. This can seen graphically in Figure 2 where, for any $\theta < \hat{\theta}_2$, conditions (LC_S) and (PC_B) cannot hold simultaneously. Therefore, at best, firm P can bring one firm to accept the license at a low royalty rate, but not both (if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$). At worst, it is unable to sell a license to any of them (if $\theta \in [\hat{\theta}_W, \min\{\hat{\theta}_1, \hat{\theta}_2\})$).

Figure 3 represents the equilibrium outcome for the linear Cournot model. On the figure, there are five different regions for the equilibrium royalty rate, as we move from weak to strong patents.

In region (I), for $\theta < \hat{\theta}_W$, the patent is weak. Firm P cannot obtain any royalty revenue, as the downstream firms always refuse to pay for the license, anticipating that it will be unwilling to bring them to court.

In region (II), where $\theta \in [\hat{\theta}_W, \hat{\theta}_1)$, firm P cannot bring even one firm to accept the license, because the litigation credibility and license acceptability constraints cannot be reconciled. So, litigation is inevitable, and firm P is better-off setting the monopoly royalty rate r^m.

In region (III), where $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$,[26] firm P has two options. First, it can charge the royalty rate such that the licensing constraint (PC_S) binds, in which case only one

[26] In the Cournot model, the case where $\hat{\theta}_1 > \hat{\theta}_2$ can occur for higher litigation costs.

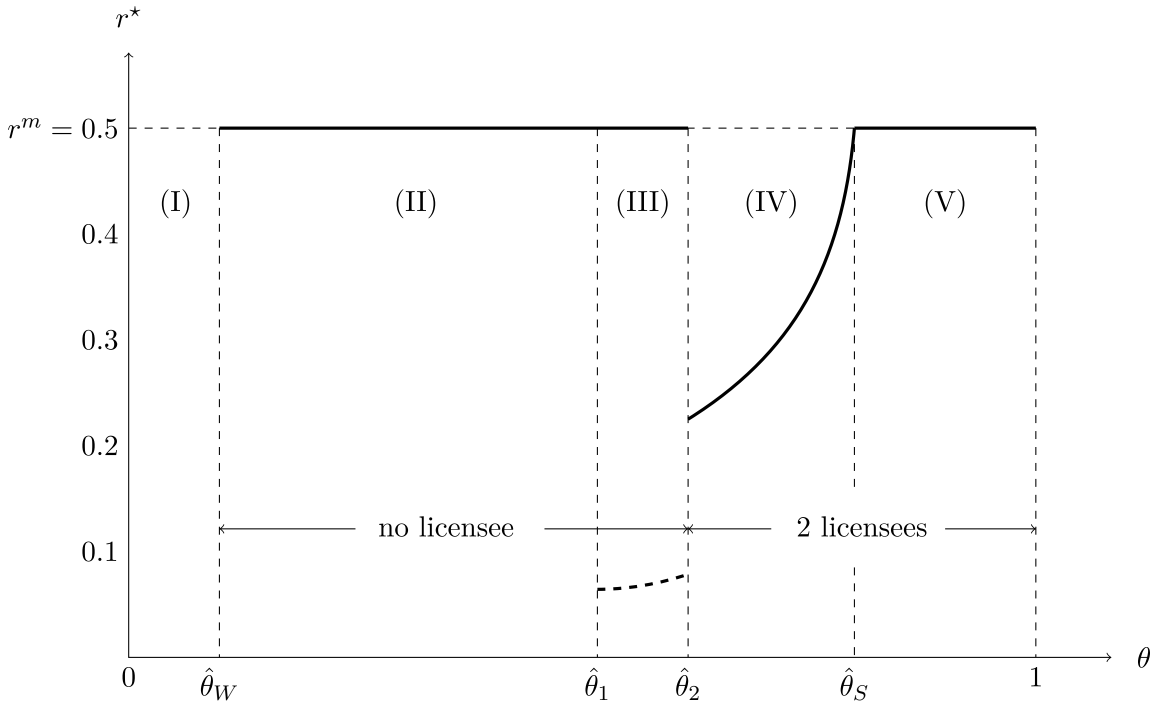

Figure 3: Equilibrium royalty rate as a function of the patent strength θ.

firm takes the license and there is no litigation; this corresponds to the thick dashed line on the figure. Alternatively, firm P can set the monopoly royalty rate r^m, which is not accepted by the firms, and go to court. In this Cournot example, we find that firm P makes a higher profit by following the latter strategy.

In region (IV), where $\theta \in [\hat{\theta}_2, \hat{\theta}_S)$, again, firm P has two options. First, it can charge the royalty rate such that the licensing constraint (PC_B) binds, in which case the two downstream firms accept the license. Or, it can ask for the monopoly royalty rate, which is not accepted by the firms, and go to court. In this example, we find that the patent holder makes a higher profit with the former strategy.

Finally, in region (V), where $\theta \geq \hat{\theta}_S$, the patent is strong and the two firms accept the license at the monopoly royalty rate r^m.

To sum up, a patent holder with a patent of intermediate strength ($\theta \in [\hat{\theta}_W, \hat{\theta}_2)$) is unable to license its technology to all downstream firms, because it faces a litigation credibility and a license acceptability constraints that cannot be satisfied at the same time. Consequently, litigation tends to occur in equilibrium for this range of patent strength.

5 Sequential licensing

In the baseline model, we have considered that downstream firms make their licensing decisions simultaneously. However, if one considers sequential licensing decisions, the licensing game yields the same outcome.[27] This can be seen in Figure 4, which shows the equilibrium of the sequential licensing game for the linear Cournot model as a function of the patent strength θ (on the horizontal axis) and the royalty rate r (on the vertical axis). Therefore, the patent holder's decision is unchanged if it approaches firms sequentially.

The only difference with the baseline game is that the sequential licensing game may yield a first or a second-mover advantage for the downstream firms. When only one firm accepts the license, and there is no litigation, the first mover can safely refuse the license and use the technology for free (and hence, benefit from a cost advantage). Conversely, when only one firm accepts the license, and there is litigation, the first mover is better off taking the license to save the litigation cost. A second-mover advantage can arise when it is a dominant strategy for the firms to refuse the license, even when they anticipate that the patent holder will go to court. In this case, with sequential litigation, the first mover refuses the license, and firm P litigates. Therefore, the second mover can observe the court's decision without incurring any litigation cost.

In what follows, we use this sequential licensing framework to study the impact on the equilibrium outcome of (i) cost asymmetries between downstream firms, and (ii) discriminatory royalties.

5.1 Cost asymmetries

So far, we have assumed that the two downstream firms were identical. Therefore, with sequential licensing, it does not matter for the patent holder which firm it approaches first. However, if the patent holder negotiates with a low-cost and a high-cost firm, we find that it faces a trade-off when considering which firm to approach first (see Appendix F for the analysis). On the one hand, licensing the high-cost firm first improves its litigation

[27] See Appendix E for the detailed analysis. In the appendix, we discuss why the equilibrium outcome is (almost) the same with simultaneous and sequential licensing.

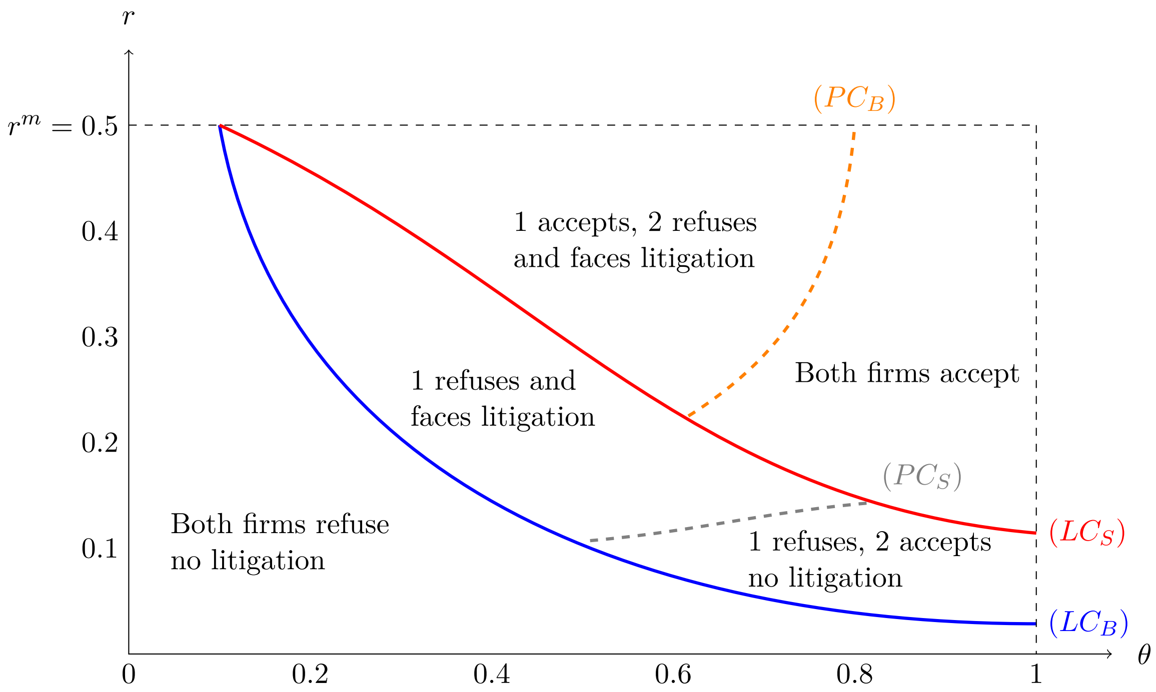

Figure 4: Equilibrium of the sequential licensing game as a function of θ and r.

credibility if the other (low-cost) firm refuses the license due to a lower opportunity cost of going to court. On the other hand, the second mover approached by the patent holder is more willing to accept the license if the low-cost firm has already agreed to pay.

If the license acceptability constraint is the binding constraint for the patent holder (which corresponds to Case (iii) of Proposition 2 in the baseline model), the patent holder should approach the low-cost firm first. Otherwise, it is ambiguous and we can find cases where the patent holder is better off dealing with the high-cost firm first (see the appendix).

5.2 Discriminatory royalties

We have assumed that the patent holder is constrained to a FRAND licensing rule. Therefore, it cannot make different offers to the two downstream firms. One may wonder whether it is this ND (non-discriminatory) component of FRAND that reduces the patent holder's ability to license its technology and triggers litigation. In this section, we thus investigate the case where the patent holder can set discriminatory royalties. We show that it only marginally improves its ability to license.

We modify the sequential licensing game by assuming that in the first stage, firm P announces royalty rates r_1 and r_2 for firm 1 and firm 2, respectively. The rest of the game follows, where firm P approaches each firm sequentially and can litigate if a firm refuses its license offer. If firm P goes to court and the patent is invalidated, the firms can use its technology for free. If the patent is held valid, we assume that each firm i has to pay the pre-announced royalty rate r_i.

In Appendix G1, we determine the equilibrium outcome of the licensing game for given royalty rates r_1 and r_2. In the first stage, firm P then chooses the royalty rates r_1 and r_2 to maximize its profit, anticipating how the downstream firms will react in the following stages. We can characterize the equilibrium of the sequential game with discriminatory royalties as follows.

Proposition 3. *Consider that firm P can make different licensing offers to firm 1 and firm 2, and approaches them sequentially.*

(i) If $\theta < \hat{\theta}_W$, no firm is licensed and there is no litigation;

(ii) If $\theta \in [\hat{\theta}_W, \hat{\theta}_2)$, either no firm is licensed and firm P has to go to court, or only one firm is licensed at the royalty rate making it indifferent between accepting and refusing the license;

(iii) If $\theta \in [\hat{\theta}_2, \hat{\theta}_S)$, either the two firms are licensed at the royalty rate $r^\star$ such that (PC_B) binds, or no firm is licensed and firm P goes to court;

(iv) If $\theta \geq \hat{\theta}_S$, the two firms are licensed at the monopoly royalty rate $r^\star = r^m$.

Proof. See Appendix G2. □

Note that, due to our assumption on the symmetry of demand functions, the royalty rates r_1 and r_2 that maximize firm P's royalty revenue, $r_1 q_1(r_1, r_2) + r_2 q_2(r_1, r_2)$, are symmetric and equal to the monopoly royalty rate r^m.

If the patent is weak, firm P cannot credibly threaten to go to court, even in the best case where it sets the monopoly royalty rates $r_1 = r_2 = r^m$. Therefore, both firms refuse the license, there is no litigation and firm P makes no profit. As in the baseline model, this happens when $\theta \leq \hat{\theta}_W$. At the other extreme, if the patent is strong enough, both firms accept the license at the monopoly royalty rates $r_1 = r_2 = r^m$. As in the baseline model, this happens when $\theta \geq \hat{\theta}_S$.

Now, consider patents of intermediate strength, with $\theta \in (\hat{\theta}_W, \hat{\theta}_S)$. If $\theta \in [\hat{\theta}_2, \hat{\theta}_S)$, firm P can bring both firms to accept the license by charging them the royalty rate $r^\star$ such that condition (PC_B) binds, as in the baseline model. Alternatively, it can set the monopoly royalty rate and go to court. In both cases, there is no benefit for the patent holder to differentiate royalty rates.

If $\theta \in [\hat{\theta}_W, \hat{\theta}_2)$, firm P can license at most one firm. If one firm is licensed, firm P can set different royalty rates for the two firms to relax the litigation and licensing constraints and be able to charge a higher royalty rate to the single licensee. Alternatively, firm P can set the monopoly royalty rates $r_1 = r_2 = r^m$ and go to court.

Figure 5 shows the equilibrium royalty rate with discriminatory royalties as a function of the patent strength θ in the Cournot example. We find that the equilibrium outcome is the same as in the baseline model. The only area where the equilibrium could differ is Region (III). In this region, firm P could set a lower royalty rate for the non-licensed firm ($r_1^\star$ on the figure) to relax the participation constraint of the licensed firm and charge it a higher royalty rate ($r_2^\star$). Alternatively, it can set the monopoly royalty rate r^m, which is not accepted by the firms, and go to court. As in the baseline model, we find that in this Cournot example, firm P makes a higher profit with the latter strategy.

Therefore, even though we allow the patent holder to charge discriminatory royalties, its ability to license its technology to downstream firms is not improved. The reason is that the two effects constraining the patent holder's licensing strategy are still at play: (i) there is still an opportunity cost when the patent holder has signed (or anticipates signing) a licensing contract with one firm, which limits its litigation credibility, and hence, its ability to license the other firm; (ii) the "hold-out" effect, which limits the downstream firms' willingness to accept the license, is still operating.

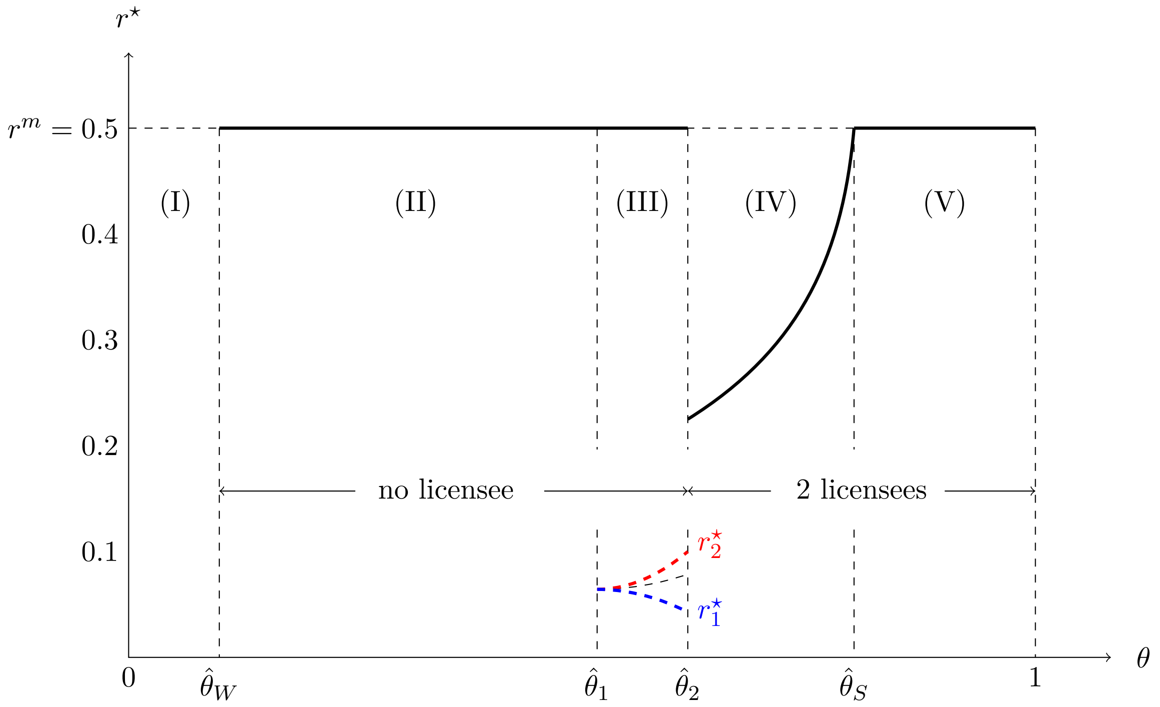

Figure 5: Equilibrium discriminatory royalty rates as a function of the patent strength θ with sequential licensing.

6 The role of injunction

We have shown that the patent holder faces two constraints that it cannot simultaneously satisfy with a patent of intermediate strength: (i) a litigation credibility constraint when one firm is already licensed (due to the opportunity cost problem); and (ii) a license acceptability constraint (due to the hold-out problem). Consequently, the holder of a patent of intermediate strength is unable to license its technology, and litigation inevitably occurs in equilibrium, as we have seen in Section 4.

However, the downstream firms would be better off if they were both licensed at the royalty rate such that condition (PC_B) binds.[28] In the Cournot example described in Figure 3, we find that the patent holder would also be better off. However, this outcome cannot be reached in equilibrium.

To restore the patent holder's ability to license its technology, we need to relax the litigation credibility constraint and/or the license acceptability constraint.

[28]Indeed, at this royalty rate, we have $\pi(r, r) = \theta\pi(r, r) + (1 - \theta)\pi(0, 0) - L \geq \theta\pi(r^m, r^m) + (1 - \theta)\pi(0, 0) - L$. Therefore, the downstream firms make a higher profit if they are both licensed at this royalty rate than if firm P sets the monopoly royalty rate and goes to court.

First, the patent holder could commit to sue any infringer, which would relax the litigation credibility constraint. For instance, Farrell and Shapiro (2008) suggest that the patent holder could make such a commitment by allowing licensees to stop paying royalties if it fails to sue an infringer. Such kinds of commitments have been implemented by the industry. For example, companies participating in the One-Blue patent pool commit to making their patents available for any enforcement action launched by the pool.[29] In our framework, if firm P can commit to litigate any infringer, the opportunity cost and cost disadvantage effects vanish, as an asymmetric outcome where one firm accepts the license, the other refuses it, and there is no litigation, cannot arise. Therefore, firm P's ability to license its technology is improved.

Another solution to improve the patent holder's ability to license its technology is to relax the constraint on the acceptability of the license by reducing the hold-out problem.

Consider that if firm P goes to court and the patent is upheld, an injunction is issued against the infringing firms. The parties then renegotiate the royalty rate in the shadow of an injunction threat. We assume that firm P has all bargaining power in this renegotiation. If both firms infringe its patent, firm P can ask the monopoly royalty rate, r^m, to both of them. If one firm is licensed at the royalty rate r, and the other is infringing, firm P can ask the latter to pay the renegotiated (profit-maximizing) royalty rate $\hat{r}(r) = \arg\max_{r'} rq_1(r, r') + r'q_2(r, r')$.[30]

We can then modify the baseline model to take into account the possibility of injunction. If both firms infringe its patent, firm P goes to court if and only if

$$2\theta r^m q(r^m, r^m) \geq L, \qquad (LC_B^I)$$

which corresponds to the litigation condition (LC_B) in the baseline model.

If one firm has accepted the license, but the other firm is infringing, firm P litigates

[29] See Peters (2011).

[30] See Appendix H for the detailed analysis of the model with injunction.

the single infringer if and only if

$$\theta\left[rq_1(r,\hat{r}(r)) + \hat{r}(r)q_2(r,\hat{r}(r))\right] \geq L + rq_1(r,0), \tag{LC_S^I}$$

which corresponds to the litigation condition (LC_S) in the baseline model.

Therefore, the possibility for the patent holder to renegotiate its licensing terms *ex-post* after an injunction order (with more favorable terms for him) improves the credibility of its litigation threat compared to the baseline case.

Furthermore, the hold-out problem vanishes, as firms can no longer expect that they will have to pay the FRAND royalty rate in the worst case (if the patent is upheld). Therefore, firms' willingness to accept the license is improved.

To see that, consider, first, that firm P's litigation threat is credible if both firms refuse the license, but not if only one firm is infringing (i.e., (LC_B^I) holds, but not (LC_S^I)). In this case, if one firm refuses the license, the other firm accepts it if and only if

$$\pi_1(r,0) \geq \theta\pi(r^m, r^m) + (1-\theta)\pi(0,0) - L, \tag{PC_S^I}$$

which corresponds to condition (PC_S) in the baseline model.

Second, consider that firm P can credibly threaten to litigate any infringer (i.e., conditions (LC_B^I) and (LC_S^I) both hold). If one firm accepts the license, the other firm follows suit if and only if

$$\pi(r,r) \geq \theta\pi_2(r,\hat{r}(r)) + (1-\theta)\pi(0,0) - L, \tag{PC_B^I}$$

which is the equivalent of condition (PC_B) in the baseline model.

In both cases, the firm's opportunity cost of accepting the license, which corresponds to its expected payoff from litigation (i.e., the right-hand side of (PC_S^I) and (PC_B^I)), is lower due to the renegotiation of licensing terms when the patent is upheld. Therefore, the threat of injunction improves the acceptability of the licensing offers.

Figure 6 shows the equilibrium outcome for the linear Cournot model as a function of the patent strength θ (on the horizontal axis) and the royalty rate r (on the vertical axis). Comparing this figure with the equivalent Figure 2 in the baseline model, one can see that the possibility of injunction relaxes the litigation credibility and license acceptability constraints for the patent holder. Therefore, there is a wider range of patent strengths and royalty rates for which firm P can achieve to sell a license to both firms.[31]

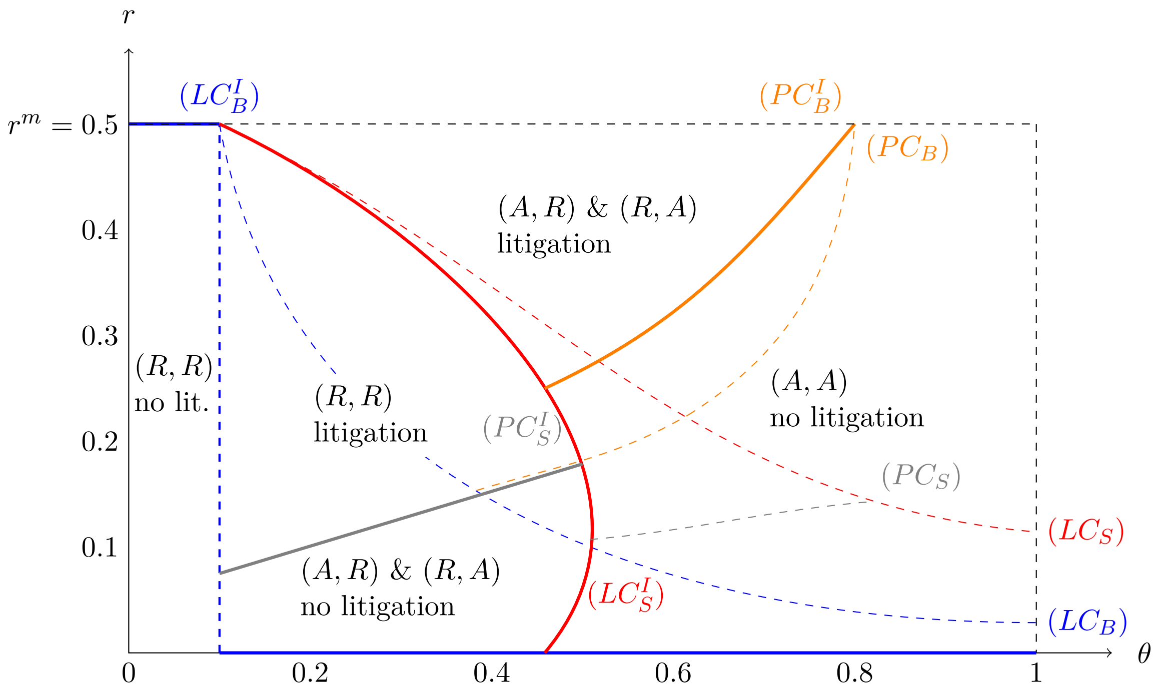

Figure 6: Equilibrium of the licensing game with injunction as a function of θ and r.

Let us define $\hat{\theta}_2^I$ as the intersection of (LC_S^I) and (PC_B^I). By construction, we have $\hat{\theta}_2^I < \hat{\theta}_2$. We can then characterize the equilibrium outcome as follows.

Proposition 4. *Consider that the court grants an injunction if the patent is upheld.*

(i) If $\theta < \hat{\theta}_W$, no firm is licensed and there is no litigation;

(ii) If $\theta \in [\hat{\theta}_W, \hat{\theta}_2^I)$, either only one firm is licensed at the royalty rate such that (PC_S^I) binds, or no firm is licensed and firm P goes to court;

(iii) If $\theta \in [\hat{\theta}_2^I, \hat{\theta}_S)$, either the two firms are licensed at the royalty rate $r^\star$ such that (PC_B^I) binds, or no firm is licensed and firm P goes to court;

[31] Note that condition (LC_S^I) holds if $\{\theta, r\}$ is located on the right side of the red curve.

(iv) If $\theta \geq \hat{\theta}_S$, the two firms are licensed at the monopoly royalty rate $r^\star = r^m$.

Proof. See Appendix H. □

With the availability of injunction, the two constraints faced by the patent holder – the litigation credibility and license acceptability constraints – are relaxed. Therefore, its ability to license its technology is improved. Since $\hat{\theta}_2^I < \hat{\theta}_2$, firm P can license its technology to both firms for a wider range of patent strengths compared to the baseline case, and at a higher royalty rate, because the downstream firms' opportunity cost of accepting a license is lower. Besides, firm P can license at least one firm in the region where it was unable to license any firm in the baseline case (i.e., region (II) in Figure 3). To do that, it sets the royalty rate such as one firm is willing to take the license if its rival refuses it (i.e., (PC_S^I) binds).

Figure 7 shows the equilibrium royalty rate with injunction (in red) as a function of the patent strength θ in the Cournot example. As a comparison, we also represent the equilibrium royalty rate in the baseline case in black. If the patent is weak ($\theta < \hat{\theta}_W$) or strong ($\theta \geq \hat{\theta}_S$), the possibility of injunction does not modify the equilibrium outcome compared to the baseline case. However, it changes the equilibrium for intermediate levels of patent strengths ($\theta \in [\hat{\theta}_W, \hat{\theta}_S)$). First, for $\theta \in [\hat{\theta}_2^I, \hat{\theta}_2]$, the two firms are licensed with the availability of injunction, compared to none in the baseline case. Second, for $\theta \in [\hat{\theta}_W, \tilde{\theta}]$, one firm is licensed, compared to none in the baseline case.[32]

In this Cournot example, the patent holder is better-off with the availability of an injunction. It is not only because its ability to license its technology is improved but also because it can charge a higher royalty rate. Overall, the downstream firms are worse-off, because they have to pay a higher royalty rate. However, they are better-off in the range of patent strengths $\theta \in [\hat{\theta}_1, \tilde{\theta}]$, where one firm is licensed with a relatively low royalty rate.

In sum, our analysis shows that the availability of injunctions improves the SEP owner's ability to license its technology by reducing the hold-out effect. However, it may

[32]For $\theta \in [\hat{\theta}_W, \hat{\theta}^I]$, firm P either sells one license at the royalty rate such that (PC_S^I) binds, or it sets the monopoly royalty rate and goes to court. In this Cournot example, we find that firm P makes a higher profit with the former strategy for $\theta \in [\hat{\theta}_W, \tilde{\theta}]$, and with the latter strategy for $\theta \in (\tilde{\theta}, \hat{\theta}^I]$.

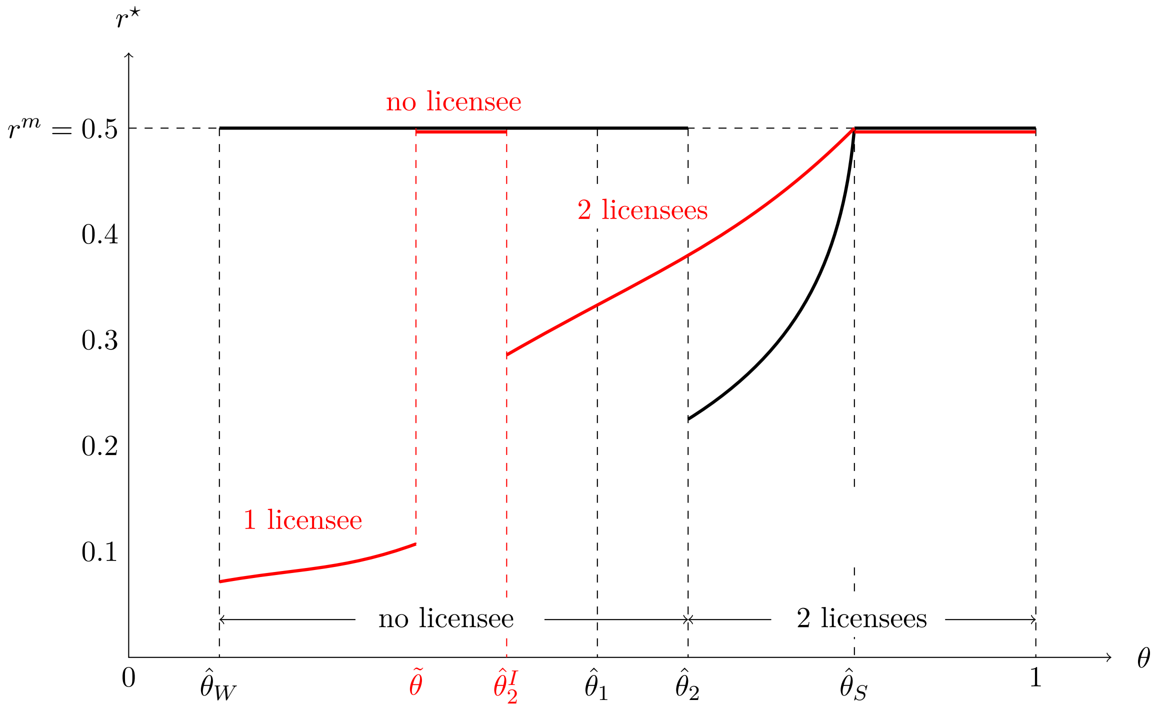

Figure 7: Equilibrium royalty rate with injunction as a function of the patent strength θ.

also lead to higher royalty rates for the downstream firms because of the patent owner's increased bargaining power, a finding in line with earlier literature on the topic (e.g., Shapiro (2010); Choi (2016)). Yet, interestingly, both the patent owner and the downstream firms may benefit from the possibility of injunction for relatively weak patents.

7 Conclusion

Licensing and enforcing standard essential patents (SEPs) is challenging, and conflicts can arise between SEP owners and implementers. In recent years, SEP holders have claimed that this is because implementers sometimes refuse to engage in good faith licensing negotiations and deliberately infringe their intellectual property rights. Some experts argue that the problem of "hold-out" by implementers is even more severe than the problem of "hold-up" by SEP owners.

In this paper, we study the licensing strategy of a SEP holder, which approaches two rival firms active in a downstream market. The SEP holder offers licenses on fair, reasonable, and non-discriminatory (FRAND) terms.

The SEP owner holds a patent of a given strength, which measures the likelihood that the patent is valid and pertinent to the standard. If the patent's strength is intermediate, we find that the patent holder is unable to license its technology to downstream firms, and litigation inevitably occurs. The patent holder's inability to license its technology is due, in particular, to the downstream firms' incentive to hold out. This is because, at worst, if the patent holder goes to court, they will have to pay the FRAND rate. At best, the patent will be invalidated, and they will be able to use the technology for free. Therefore, our paper provides a theoretical background to the idea developed in the policy debate that hold-out can be a source of litigation for SEPs.

Our results are robust to whether the SEP holder approaches downstream firms simultaneously or sequentially. They are also robust when we allow the SEP holder to offer discriminatory royalties.

Litigation is an inefficient outcome for the patent holder but also for the downstream firms. We show that allowing the SEP holder to seek an injunction against infringing firms improves its ability to license its technology. However, an injunction increases the market power of the SEP holder, and thus may harm downstream firms through an inflated royalty rate.

In our setting, we adopt a loose definition of the reasonable part of the FRAND commitment. However, we do not think that introducing uncertainty about the FRAND nature of the royalty rate proposed by the SEP holder, in the spirit of Choi (2016), for instance, would fundamentally change our findings. We also abstract from the hold-up problem. To determine the desirable FRAND rate, one would need to consider both hold-up and hold-out in a unified framework. We leave these potentially interesting extensions to future research.

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Appendix

Appendix A: Proof of Lemma 2.

If neither (LC_B) nor (LC_S) holds (i.e., $\theta < \underline{\theta}(r)$), litigation is not a credible threat in any subgame. In this case, it is a dominant strategy for a firm to refuse the license.

Now, consider the case where (LC_B) holds, but not (LC_S), which means that $\theta \in [\underline{\theta}(r), \overline{\theta}(r))$. If firm $j \neq i$ accepts the license, there is no risk for firm i of being brought to court when refusing the license. So, firm i's best response is to refuse it. If firm j refuses the license, firm i's best response is to accept it if (PC_S) holds, and to refuse it otherwise.

Finally, consider the case where both (LC_B) and (LC_S) hold, that is, $\theta \geq \overline{\theta}(r)$. If firm $j \neq i$ accepts the license, firm i's best response is to accept it if (PC_B) holds, and to refuse it otherwise. If firm j refuses the license, firm i's best response is to accept it. Indeed, firm P will go to court since one firm refused the license, and the patent's validity will be revealed for all firms. Therefore, firm i's best response is to accept the license to save on litigation costs.

Appendix B: Cournot example.

We assume that firms compete in quantities with homogenous products. The inverse demand is given by $P(Q) = 1 - Q$, where Q represents the total quantity.

The maximum ("reasonable") royalty rate that ensures non-negative quantities for the licensees is then $r = 1$ if both downstream firms are licensed, and $r = 1/2$ if only one of them is. The maximum royalty is lower in the latter case, because the licensee operates at a cost disadvantage. The monopoly royalty rate is $r^m = 1/2$.[33]

We assume that $L < 1/6$, which ensures that litigation is a credible threat if the patent is ironclad ($\theta = 1$) and the patent holder sets the monopoly royalty rate r^m.

[33]Thus, if the royalty rate is set at the monopoly level, a single licensee makes no profit.

Litigation conditions. The litigation condition (LC_B) can be expressed as $2\theta r(1-r)/3 \geq L$, which is equivalent to

$$r \geq \frac{1}{2} - \frac{\sqrt{1-6L/\theta}}{2} \equiv r_L^B(\theta),$$

where $r_L^B(\theta)$ is decreasing in θ.

The litigation condition (LC_S) can be written as $r\left[2r(1-\theta)+2\theta-1\right]/3 \geq L$, which is equivalent to

$$r \geq \frac{1-2\theta+\sqrt{4\theta^2-4\theta-24L\theta+24L+1}}{4(1-\theta)} \equiv r_L^S(\theta).$$

Using the implicit function theorem on condition (LC_S) as expressed above shows that $r_L^S(\theta)$ is decreasing in θ. Since condition (LC_S) implies condition (LC_B), we have $r_L^S(\theta) \geq r_L^B(\theta)$.

Licensing conditions. Condition (PC_S) can be expressed as $r\left[(4-\theta)r-2(2-\theta)\right]/9+L \geq 0$, which is equivalent to

$$r \leq \frac{2-\theta-\sqrt{\theta^2+9L\theta-36L+4(1-\theta)}}{4-\theta} \equiv r_P^S(\theta).$$

Using the implicit function theorem on condition (PC_S) as expressed above shows that $r_P^S(\theta)$ is increasing in θ.

Condition (PC_B) is given by $r\left[(1-\theta)r-2(1-\theta)\right]/9+L \geq 0$, which is equivalent to

$$r \leq 1 - \frac{\sqrt{\theta^2-2\theta+9L\theta-9L+1}}{1-\theta} \equiv r_P^B(\theta).$$

Using the implicit function theorem on condition (PC_B) as expressed above shows that $r_P^B(\theta)$ is increasing in θ. Since condition (PC_S) implies condition (PC_B), we have $r_P^B(\theta) \geq r_P^S(\theta)$.

Appendix C: Variations of litigation and licensing conditions.

We study the variations of the litigation and licensing conditions with respect to the royalty rate r for the general model.

Condition (LC_B)**.** Condition (LC_B) holds if and only if $\theta \geq \underline{\theta}(r)$, with $\underline{\theta}(r) = L/(2rq(r,r))$. Since $2rq(r,r)$ is quasi-concave and maximized at r^m, then $\underline{\theta}(r)$ is decreasing in r for $r \in [0, r^m]$. We can thus define the inverse function, $r_L^B(\theta)$, which is decreasing in θ, such that condition (LC_B) holds if and only if $r \geq r_L^B(\theta)$.

Condition (LC_S)**.** Condition (LC_S) holds if and only if $\theta \geq \overline{\theta}(r)$, with

$$\overline{\theta}(r) = \frac{L + rq_1(r,0)}{2rq(r,r)}.$$

Let $\Pi^m(r) \equiv 2rq(r,r)$ represent firm P's royalty revenues if both firms pay for the license, and $\Pi^s(r) \equiv rq_1(r,0)$ its revenues if a single firm (here, firm 1, without loss of generality) does. The threshold $\overline{\theta}(r)$ can then be written as $\overline{\theta}(r) = (L + \Pi^s(r))/\Pi^m(r)$. A sufficient condition for $d\overline{\theta}(r)/dr \leq 0$ to hold is that

$$\frac{d\Pi^s(r)/dr}{\Pi^s(r)} \leq \frac{d\Pi^m(r)/dr}{\Pi^m(r)}. \tag{1}$$

Let $\epsilon_m \equiv -q'(r,r)r/q(r,r)$ and $\epsilon_s \equiv -q_1'(r,0)r/q_1(r,0)$ represent the elasticity of equilibrium demand to the royalty rate r when both firms are licensed and when a single one is, respectively. Then, condition (1) holds if and only if $\epsilon_s \geq \epsilon_m$, that is, if the demand with a single licensee is more elastic to the royalty rate than the demand with two licensees.

If $\epsilon_s \geq \epsilon_m$, $\overline{\theta}(r)$ is decreasing in r. We can then define the inverse function $r_L^S(\theta)$, which is decreasing in θ, such that condition (LC_S) holds if and only if $r \geq r_L^S(\theta)$.

Condition (PC_S)**.** Condition (PC_S) holds if $G(r) \equiv -\theta\pi(r,r) + \pi_1(r,0) - (1-\theta)\pi(0,0) + L \geq 0$. We have $G(0) = L > 0$ and $\lim_{r\to\infty} G(r) = -(1-\theta)\pi(0,0) + L$. Furthermore, we have $G'(r) \leq 0$ if and only if $-\theta\partial\pi(r,r)/\partial r + \partial\pi_1(r,0)/\partial r \leq 0$. For this condition to hold for all $\theta \in (0,1)$, a sufficient condition is that $\partial\pi(r,r)/\partial r \geq \partial\pi_1(r,0)/\partial r$ (where both derivatives are negative). In words, if the licensee's profits decrease faster with the royalty rate when only one producer pays for the license than if they both do, then $G(r)$ is decreasing in r. In this case, there is a unique threshold, $r_P^S(\theta)$, such that condition (PC_S) holds if and only if $r \leq r_P^S(\theta)$, with $r_P^S(\theta)$ increasing in θ.

Condition (PC_B). Condition (PC_B) is equivalent to $\pi(r,r) - \pi(0,0) + L/(1-\theta) \geq 0$. If $\pi(r,r)$ is decreasing in r, there is a unique threshold, $r_P^B(\theta)$, such that condition (PC_B) holds if and only if $r \leq r_P^B(\theta)$, with $r_P^B(\theta)$ increasing in θ.

Appendix D: Proof of Proposition 2.

We solve for the equilibrium royalty rate for the different relevant ranges of the patent strength θ.

(1) First, consider the case where $\theta < \hat{\theta}_W \equiv \underline{\theta}(r^m)$. Since the monopoly royalty revenues $2rq(r,r)$ are quasi-concave, we have $\theta \leq \underline{\theta}(r)$ for any $\theta < \hat{\theta}_W$ and any $r \in [0, r^m]$. Therefore, $\Pi^P = 0$ for all $r \in [0, r^m]$.

(2) If $\theta \in [\hat{\theta}_W, \min\{\hat{\theta}_1, \hat{\theta}_2\})$, there are three possible outcomes to the licensing game. First, if $\theta < \underline{\theta}(r)$, firm P makes no profit. Second, if $\theta \in [\underline{\theta}(r), \overline{\theta}(r))$, the two firms refuse the license offer and firm P goes to court. Finally, if $\theta \geq \overline{\theta}(r)$, one firm accepts the license, the other refuses it and firm P goes to court. Therefore, since it either makes no royalty revenues or has to go to court, the profit-maximizing strategy for firm P is to set the monopoly royalty rate $r^\star = r^m$ and to go to court.

(3) The case where $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$ happens if $\hat{\theta}_1 < \hat{\theta}_2$. In this case, there are four possible outcomes to the licensing game. If $\theta < \underline{\theta}(r)$, firm P makes no profit. If $\theta \in [\underline{\theta}(r), \overline{\theta}(r))$, firm P either sells the license to a single firm (if (PC_S) holds) or there is litigation (if (PC_S) does not hold). Finally, if $\theta \geq \overline{\theta}(r)$, a single firm accepts the license and there is litigation. Therefore, firm P either sets $r^\star$ such that (PC_S) binds and only one license is sold, or $r^\star = r^m$ and there is litigation.

(4) If $\theta \in [\hat{\theta}_2, \hat{\theta}_S)$, there are five possible outcomes to the licensing game. If $\theta < \underline{\theta}(r)$, firm P makes no profit. If $\theta \in [\underline{\theta}(r), \overline{\theta}(r))$, firm P either sells the license to a single firm (if (PC_S) holds) or there is litigation (if (PC_S) does not hold). Finally, if $\theta \geq \overline{\theta}(r)$, both firms accept the license if (PC_B) holds; otherwise, there is litigation. Therefore, the profit-maximizing strategy for firm P is either to set the royalty rate such that (PC_B) binds, in which case the two firms take a license and there is no litigation, or to set $r^\star = r^m$ and go to court.

(5) Finally, if $\theta \geq \hat{\theta}_S$, since the firms accept the license at the monopoly royalty rate, firm P sets $r^\star = r^m$ and there is no litigation.

Appendix E: Sequential licensing game.

We consider the following sequential game, where (without loss of generality) firm P approaches firm 1 first and then firm 2:

1. The patent holder, firm P, sets the per-unit royalty rate, r.
2. Firm P offers a license to firm 1, which decides whether to accept it.
3. If firm 1 has refused the license, firm P decides whether to litigate. In case of litigation, the patent is either validated or invalidated by the court.
4. Firm P offers a license to firm 2, and firm 2 decides whether to accept it.
5. If firm 2 has refused the license, firm P decides whether to litigate.
6. Profits are realized.

We look for the Subgame Perfect Equilibrium of this game.

As in the baseline model, we assume that if the patent is held invalid by the court, downstream firms are free to produce without paying the license fee, even if they previously accepted the license.

Conversely, if the patent is held valid, we assume that the downstream firms respect the patent and pay the license fee. Indeed, if the patent is validated in the first trial against firm 1, under the collateral estoppel (or issue preclusion) doctrine, the second trial against firm 2 would only deal with the matter of infringement and the litigation costs of both parties will be lower than in the first trial. Moreover, if both firms use the same technology, as it is the case in our setting, under the same doctrine, firm 2 would be automatically considered as an infringer in a second trial, which implies that both parties' litigation costs would essentially drop to zero.

Stage 5: P's litigation decision against firm 2

At Stage 5, firm 2 has refused the license and firm P decides whether to litigate. Firm P's decision depends on whether firm 1 has previously accepted or refused the license.

If firm 1 refused the license, both firms infringe P's patent and it goes to court if and only if (LC_B) holds. If firm 1 accepted the license, one firm has accepted the license, the other refused it. So, P sues firm 2 if and only if (LC_S) holds. Therefore, the analysis is similar to the baseline model, and Lemma 1 applies. Note that there is a third possible subgame, where firm 1 refused the license and P went to court. In this case, the patent has been either validated, in which case both firms have to pay the license fee, or invalidated, and they can both use the technology for free.

Stage 4: Firm 2's licensing decision

At Stage 4, firm 2 decides whether to accept the license, anticipating firm P's litigation decision at the next stage. If firm 1 refused the license and was not sued, firm 2 accepts the license if condition (LC_B) holds (i.e., litigation is a credible threat if firm 2 turns down the offer) as well as condition (PC_S); otherwise, firm 2 refuses the license. If firm 1 accepted the license, firm 2 accepts the license if condition (LC_S) holds (litigation is a credible threat if firm 2 refuses the license) as well as condition (PC_B); otherwise, firm 2 refuses the license.

Stage 3: P's litigation decision against firm 1

To determine firm P's optimal litigation decision at Stage 3, we compare its expected profit if it litigates firm 1 and if it does not.

If firm P litigates firm 1, the patent's validity is revealed in court. Firm P's expected profit is then $\Pi_P = 2\theta rq(r,r) - L$, which is positive if and only if condition (LC_B) holds.

If firm P does not litigate, from our analysis of Stage 4 and Stage 5, we know that if condition (LC_B) does not hold, firm P cannot credibly threaten firm 2 to go to court. Therefore, firm 2 refuses the license, and firm P makes no profit ($\Pi_P = 0$).

If (LC_B) holds, firm P can credibly threaten firm 2 to go to court. If condition (PC_S) holds, firm 2 accepts the license, and firm P's profit is $\Pi_P = rq_2(0,r)$. If (PC_S) does not hold, firm 2 refuses the license, and firm P goes to court; firm P's expected profit is then $\Pi_P = 2\theta rq(r,r) - L$.

We can now determine firm P's optimal litigation decision by comparing its expected profit if it sues firm 1 and if it does not.

If condition (LC_B) does not hold, firm P does not sue firm 1, because its expected profit if it goes to court is negative, and it cannot credibly threaten firm 2 of litigation either. Therefore, the two downstream firms use the technology without paying the license fee, and firm P makes no profit ($\Pi_P = 0$).

If condition (LC_B) holds, firm P's litigation decision depends on whether firm 2 is willing to take a license if firm 1 does not take one and firm P does not sue.

If this is the case, i.e., condition (PC_S) holds, firm P litigates firm 1 iff $2\theta rq(r,r) - L \geq rq_2(0,r)$. Note that this condition is equivalent to the litigation condition (LC_S), since under our symmetry assumptions we have $q_2(0,r) = q_1(r,0)$. Firm P trades off the royalty revenues it can obtain from both downstream firms with some probability (against litigation costs) and the royalty revenues it can derive for sure from a single producer.

If (PC_S) does not hold, firm P is indifferent between litigating firm 1 at Stage 3 and firm 2 at Stage 5. We assume that, in this case, it chooses to litigate firm 1.

Summing up, if firm 1 refused the license, P goes to court if condition (LC_B) holds and either (i) condition (PC_S) does not hold; or (ii) condition (PC_S) holds as well as the litigation condition (LC_S).

Stage 2: Firm 1's licensing decision

We solve for firm 1's optimal licensing decision in the three possible cases: (i) $\theta < \underline{\theta}(r)$; (ii) $\underline{\theta}(r) \leq \theta < \overline{\theta}(r)$; and (iii) $\theta \geq \overline{\theta}(r)$. We show that firm 1 accepts the license offer if $\theta \geq \overline{\theta}(r)$, and refuses it otherwise.

(i) If $\theta < \underline{\theta}(r)$, firm P is unable to enforce its patent rights in any situation. As (LC_B) does not hold, from the analysis of Stage 3, we know that firm P will not litigate if firm 1 refuses the license. Therefore, firm 1 refuses the license and there is no litigation. Since (LC_B) does not hold, firm 2 also refuses the license at Stage 4 and firm P makes no profit.

(ii) If $\theta \in [\underline{\theta}(r), \overline{\theta}(r))$, firm P can credibly threaten to litigate if no firm accepts the license, but its threat is not credible if one firm accepts it.

Consider first that firm 1 accepts the license. Since firm P does not litigate firm 2 if it refuses the license (as (LC_S) does not hold), firm 2 refuses it and firm 1's profit is $\pi_1 = \pi_1(r, 0)$.

Now, consider that firm 1 refuses the license. Since (LC_B) holds, firm P litigates firm 2 if it also refuses the license. Therefore, firm 2 accepts the license if and only if condition (PC_S) holds. If (PC_S) holds, firm 2 accepts the license and firm P does not litigate firm 1 since the litigation condition (LC_S) does not hold. Firm 1's profit is then $\pi_1 = \pi_1(0, r)$. If (PC_S) does not hold, firm P litigates firm 1, and firm 1's expected profit is $\pi_1 = \theta\pi(r, r) + (1 - \theta)\pi(0, 0) - L$.

Summing up, if (PC_S) holds, firm 1 refuses the license, as $\pi_1(0, r) > \pi_1(r, 0)$ under our assumptions. If (PC_S) does not hold, firm 1 accepts the license iff $\pi_1(r, 0) \geq \theta\pi(r, r) + (1 - \theta)\pi(0, 0) - L$. Since $\pi_1(r, 0) = \pi_2(0, r)$, this condition is equivalent to condition (PC_S). Therefore, it does not hold, meaning that firm 1 refuses the license in this case too. Thus, if $\theta \in [\underline{\theta}(r), \overline{\theta}(r))$, firm 1 refuses the license, and firm 2 accepts it if and only if (PC_S) holds.

(iii) Finally, if $\theta \geq \overline{\theta}(r)$, firm P can credibly threaten to go to court against an infringer in any situation. If firm 1 refuses the license, since (LC_B) and (LC_S) hold, firm P goes to court, and firm 1's expected profit is $\pi_1 = \theta\pi(r,r) + (1-\theta)\pi(0,0) - L$.

Now, consider that firm 1 accepts the license. Since (LC_S) holds, firm P goes to court if firm 2 refuses the license. Therefore, firm 2 accepts the license if condition (PC_B) holds, in which case firm 1 makes the profit $\pi_1 = \pi(r,r)$. If (PC_B) does not hold, firm 2 refuses the license and firm P sues firm 2; firm 1's expected profit is then $\pi_1 = \theta\pi(r,r) + (1-\theta)\pi(0,0)$.

Summing up, if (PC_B) holds, firm 1 accepts the license iff $\pi(r,r) \geq \theta\pi(r,r) + (1-\theta)\pi(0,0) - L$. Since this inequality is equivalent to condition (PC_B), it means that firm 1 accepts the license. If (PC_B) does not hold, firm 1 makes a higher expected profit by accepting than by refusing the license. Indeed, in all cases, firm P goes to court. However, if firm 1 accepts the license, it is its rival, firm 2, which bears the litigation cost.

Stage 1: P's royalty decision

Using the analysis of Stages 2-5, we can characterize the equilibrium of the sequential licensing game for a given royalty rate r.

Proposition 5. *For a given royalty rate r, the equilibrium outcome of the sequential licensing game is as follows:*

(a) If $\theta < \underline{\theta}(r)$, firm 1 and firm 2 refuse the license; there is no litigation and firm P makes no profit.

(b) If $\underline{\theta}(r) \leq \theta < \overline{\theta}(r)$, firm 1 refuses the license. If (PC_S) holds, firm 2 accepts the license and there is no litigation. Otherwise, if (PC_S) does not hold, firm P litigates firm 1.

(c) If $\theta \geq \overline{\theta}(r)$, firm 1 accepts the license. Firm 2 follows suit if (PC_B) holds; otherwise, firm 2 refuses the license and firm P litigates firm 2.

Comparing Proposition 1 and Proposition 5 shows that since the downstream firms are identical, the equilibrium outcome is the same for the patent holder for a given patent strength θ and royalty rate r. Therefore, the patent holder's choice of royalty rate, as described in Proposition 2, is also the same in the two settings.

To understand why the equilibrium with simultaneous and sequential licensing is the same, first note that firm 2's best response in the sequential game is the same as the best response of a downstream firm in the simultaneous game.

Therefore, if the equilibrium outcome in the simultaneous game is asymmetric (i.e., one firm accepts the license and the other refuses it), the sequentiality of moves allows firm 1 to pick which equilibrium is played (i.e., whether it is the firm accepting or refusing the license), but it does not otherwise change the equilibrium outcome.

Now, consider the possible symmetric equilibria in the simultaneous game. If condition (LC_B) does not hold, it is a dominant strategy for a firm to refuse the license in any situation, since litigation is not credible. Therefore, both firms refuse the license, whether licensing takes place simultaneously or sequentially. If (LC_S) and (PC_B) hold, both firms prefer to be licensed than to go to court; therefore, irrespective of the order of moves, they both accept the license. Finally, if (LC_B) holds but not (LC_S), and (PC_S) does not hold, it is a dominant strategy for firm 2 to refuse the license. Since (PC_S) does not hold, firm 1 is also better off refusing the license, and litigation occurs.

Appendix F: Cost asymmetries.

We study the case where one firm has a lower marginal cost than its rival. To simplify the analysis, we assume a small asymmetry in marginal costs: firm 1 has a zero marginal cost, whereas firm 2 has a marginal equal to $\epsilon > 0$, where ϵ is small. We study how the asymmetry in marginal costs affects the equilibrium of the sequential licensing game for a given royalty rate r.

At the last stage of the game, if both firms have refused the license, firm P goes to court if and only if

$$\theta \left[q_1(r, r+\epsilon) + q_2(r, r+\epsilon)\right] r \geq L, \qquad (LB_B^\epsilon)$$

which corresponds to condition (LC_B) in the baseline model. Note that due to the symmetry of demand functions, condition (LB_B^ϵ) is the same whether firm P approaches firm 1 or firm 2 first.

If the first firm that firm P has approached, firm i, has accepted the license, and the other firm $j \neq i$ refuses it, firm P goes to court if and only if

$$\theta\left[q_1(r, r+\epsilon)+q_2(r, r+\epsilon)\right] r \geq L+R_i^S, \tag{LC_S^ϵ}$$

which corresponds to condition (LC_S) in the baseline model. In (LC_S^ϵ), the term R_i^S represents the royalties derived from firm i if it is the only licensee, with $R_1^S = q_1(r,\epsilon)r$ if firm P approaches firm 1 first, and $R_2^S = q_2(0, r+\epsilon)r = q_1(r+\epsilon, 0)r$ if it approaches firm 2 first. Since $q_1(r,\epsilon) > q_1(r+\epsilon, 0)$, the opportunity cost of going to court, which is equal to R_i^S, is higher if firm P approaches firm 1 first. Therefore, its litigation threat against a single infringer is stronger if it approaches the high-cost firm first.

Consider now the licensing decision of the second mover and assume that the litigation conditions hold. If the first mover, firm i, has refused the license and firm P did not sue, the second mover, firm j, accepts the license if and only if

$$\pi_j(r, r+\epsilon) \geq \pi_j(0, \epsilon)-\frac{L}{1-\theta}+\frac{\pi_j(r, r+\epsilon)-\pi_j^S}{1-\theta}, \tag{PC_S^ϵ}$$

which corresponds to condition (PC_S) in the baseline model. We have $\pi_1^S = \pi_1(r,\epsilon)$ if firm 1 is the second mover, and $\pi_2^S = \pi_2(0, r+\epsilon)$ if it is firm 2, with $\pi_1(r,\epsilon) > \pi_2(0, r+\epsilon)$.

Finally, if the first mover has accepted the license, the second mover accepts the license if and only if

$$\pi_j(r, r+\epsilon) \geq \pi_j(0, \epsilon)-\frac{L}{1-\theta}, \tag{PC_B^ϵ}$$

which corresponds to condition (PC_B) in the baseline model.

In the Cournot model, we find that the licensing conditions (PC_S^ϵ) and (PC_B^ϵ) are more easily met if firm P approaches the low-cost firm, firm 1, first.

Therefore, when considering whether to approach the low-cost or the high-cost firm first, firm P faces a trade-off. On the one hand, licensing the high-cost firm first improves its litigation credibility in case the other (low-cost) firm refuses the license, due to a lower opportunity cost of going to court. On the other hand, the second mover approached by firm P is more willing to accept the license if the low-cost firm has already agreed to pay.

If firm P owns a strong patent, its licensing strategy is constrained by condition (PC_B^{ϵ}). In this case, firm P makes a higher profit by approaching the low-cost firm first. Otherwise, if firm P's patent is of intermediate strength, the two effects discussed above are at play, and it is more ambiguous which firm should firm P approach first. For example, in the Cournot model, for $L = 0.02$ and $\epsilon = 0.1$, we find that firm P makes a higher profit by approaching the low-cost firm first if $\theta \in (0.54, 0.77)$, and by approaching the high-cost firm first if $\theta \in (0.52, 0.54)$.

Appendix G: Discriminatory royalties.

G1: Equilibrium of the licensing game

We allow firm P to set different royalties for firm 1 and firm 2, and assume that it publicly commits to these royalty rates at the beginning of the game. The game then unfolds as follows:

1. Firm P sets the per-unit royalty rates r_1 and r_2 for firm 1 and firm 2, respectively.
2. Firm P offers a license to firm 1 at the royalty rate r_1. Firm 1 decides whether to accept it.
3. If firm 1 has refused the license, firm P decides whether to litigate firm 1.
4. Firm P offers a license to firm 2 at the royalty rate r_2. Firm 2 decides whether to accept it.
5. If firm 2 has refused the license, firm P decides whether to litigate firm 2.
6. Profits are realized.

If the patent is invalidated, the firms can use firm P's technology for free. If it is held valid, each firm i has to pay the pre-announced royalty rate r_i. We solve for the equilibrium of this sequential licensing game for given royalty rates r_1 and r_2.

Stage 5: Decision to litigate firm 2. Assume that firm 2 has refused the license. If firm 1 refused the license and firm P did not sue, firm P sues firm 2 if and only if

$$\theta\left[r_1 q_1(r_1, r_2) + r_2 q_2(r_1, r_2)\right] \geq L. \qquad (LC_B^D)$$

If firm 1 accepted the license, firm P sues firm 2 if and only if

$$\theta\left[r_1 q_1(r_1, r_2) + r_2 q_2(r_1, r_2)\right] \geq L + r q_1(r_1, 0) \qquad (LC_S^D)$$

Stage 4: Firm 2's licensing decision. If firm 1 refused the license and firm P did not sue, firm 2 accepts the license if (LC_B^D) holds and $\pi_2(0, r_2) \geq \theta\pi_2(r_1, r_2) + (1-\theta)\pi(0, 0) - L$, that is, if

$$\pi_2(r_1, r_2) \geq \pi(0, 0) - \frac{L}{1-\theta} + \frac{\pi_2(r_1, r_2) - \pi_2(0, r_2)}{1-\theta}. \qquad (PC_S^D)$$

If firm 1 accepted the license, firm 2 follows suit if (LC_S^D) holds and $\pi_2(r_1, r_2) \geq \theta\pi_2(r_1, r_2) + (1-\theta)\pi(0, 0) - L$, that is, if

$$\pi_2(r_1, r_2) \geq \pi(0, 0) - \frac{L}{1-\theta}. \qquad (PC_B^D)$$

Stage 3: Decision to litigate firm 1. To determine firm P's optimal litigation decision regarding firm 1, we compare its expected profits if it goes to court and if it does not. If firm P litigates firm 1, it obtains the expected profit $\theta\left[r_1 q_1(r_1, r_2) + r_2 q_2(r_1, r_2)\right] - L$, which is positive if and only (LC_B^D) holds.

Now, consider the case where firm P does not litigate firm 1. If (LC_B^D) does not hold, firm P cannot credibly threaten firm 2 to go to court. Therefore, firm 2 refuses the license and firm P makes no profit. If (LC_B^D) holds and (PC_S^D) holds too, firm 2 accepts the license and firm P makes the profit $r_2 q_2(0, r_2)$. If (LC_B^D) holds but not (PC_S^D), firm 2 refuses the license and firm P goes to court, obtaining the expected profit $\theta\left[r_1 q_1(r_1, r_2) + r_2 q_2(r_1, r_2)\right] - L$.

To sum up, if (LC_B^D) does not hold, firm P does not litigate firm 1 and makes no profit. If (LC_B^D) holds but not (PC_S^D), firm 2 will refuse the license if firm P does not sue firm 1. Therefore, firm P litigates either firm 1 or firm 2, obtaining in both cases the expected profit $\theta\left[r_1q_1(r_1,r_2)+r_2q_2(r_1,r_2)\right]-L$. As in the baseline model, we assume that firm P then sues firm 1. Finally, if (LC_B^D) and (PC_S^D) hold, firm P litigates firm 1 if and only if

$$\theta\left[r_1q_1(r_1,r_2)+r_2q_2(r_1,r_2)\right]\geq L+r_2q_2(0,r_2).$$

Note that contrary to the baseline model, this condition is not equivalent to (LC_S^D), as $r_2q_2(0,r_2)$ can be different from $r_1q_1(r_1,0)$.

Stage 2: Firm 1's licensing decision. We can now determine firm 1's optimal licensing decision. First, if $\theta(r_1q_1+r_2q_2)<L$, firm P's litigation threat is not credible, and hence, firm 1 refuses the license (and firm 2 does the same).

Second, assume that $L\leq\theta(r_1q_1+r_2q_2)<L+r_1q_1(r_1,0)$. Firm P can credibly threaten to go to court if no firm accepts the license. However, if firm 1 takes the license, firm P cannot credibly threaten to go to court when negotiating with firm 2 due to the opportunity cost effect. Therefore, if firm 1 accepts the license, it is the only licensee, and it earns the profit $\pi_1(r_1,0)$. If firm 1 refuses the license, firm 2 accepts the license if (PC_S^D) holds, in which case firm 1 obtains the profit $\pi_1(0,r_2)$. If (PC_S^D) does not hold, firm 2 refuses the license and firm P sues firm 1. In this case, firm 1's expected profit is $\theta\pi_1(r_1,r_2)+(1-\theta)\pi(0,0)-L$.

Summing up, if (PC_S^D) holds, firm 1 refuses the license as $\pi_1(r_1,0)<\pi_1(0,r_2)$ for $r_1,r_2>0$, and firm 2 accepts it. If (PC_S^D) does not hold, firm 1 accepts the license if $\pi_1(r_1,0)\geq\theta\pi_1(r_1,r_2)+(1-\theta)\pi(0,0)-L$. Note that this condition is not equivalent to (PC_S^D), as r_1 and r_2 can be different.

Third, and finally, assume that $\theta(r_1q_1+r_2q_2)\geq L+r_1q_1(r_1,0)$. If firm 1 refuses the license, firm P goes to court, and firm 1 obtains the profit $\theta\pi_1(r_1,r_2)+(1-\theta)\pi(0,0)-L$. Now, consider that firm 1 accepts the license. If (PC_B^D) holds (i.e., firm 2 follows suit), then firm 1 makes the profit $\pi_1(r_1,r_2)$. If (PC_B^D) does not hold (i.e., firm 2 refuses the license), firm P goes to court against firm 2 and firm 1 obtains the profit $\theta\pi_1(r_1,r_2)+(1-\theta)\pi(0,0)$.

Summing up, if (PC_B^D) holds, firm 1 accepts the license if $\pi_1(r_1, r_2) \geq \theta\pi_1(r_1, r_2) + (1-\theta)\pi(0,0) - L$. Note that this condition is not equivalent to (PC_B^D), as r_1 and r_2 can be different. If (PC_B^D) does not hold, firm 1 accepts the license to avoid litigation costs.

Using the previous analysis, we can characterize the equilibrium of the sequential licensing game for given royalty rates r_1 and r_2. To simplify the exposition, we drop the arguments of $q_1(r_1, r_2)$ and $q_2(r_1, r_2)$.

Lemma 3. *Consider that firm P offers royalty rates r_1 and r_2 to firm 1 and firm 2, respectively. The equilibrium of the sequential licensing game is then as follows:*

1. *If $\theta(r_1q_1 + r_2q_2) < L$, firm 1 and firm 2 refuse the license, there is no litigation and firm P makes no profit.*
2. *If $L \leq \theta(r_1q_1 + r_2q_2) < L + r_1q_1(r_1, 0)$:*
    - *If $\pi_2(0, r_2) \geq \theta\pi_2(r_1, r_2) + (1-\theta)\pi(0,0) - L$, firm 1 refuses the license and firm 2 accepts it; there is no litigation and firm P makes the profit $r_2q_2(0, r_2)$;*
    - *Otherwise, if $\pi_1(r_1, 0) \geq \theta\pi_1(r_1, r_2) + (1-\theta)\pi(0,0) - L$, firm 1 accepts the license, firm 2 refuses it, and firm P makes the profit $r_1q_1(r_1, 0)$; if $\pi_1(r_1, 0) < \theta\pi_1(r_1, r_2) + (1-\theta)\pi(0,0) - L$, firm 1 refuses the license and is sued.*
3. *If $\theta(r_1q_1 + r_2q_2) \geq L + r_1q_1(r_1, 0)$, both firms accept the license if $\pi_i(r_1, r_2) \geq \theta\pi_i(r_1, r_2) + (1-\theta)\pi(0,0) - L$ for $i = 1, 2$; otherwise, at least one firm refuses the license and firm P litigates.*

G2: Proof of Proposition 3.

In the first stage, firm P chooses its royalty rates r_1 and r_2 to maximize its profit, anticipating how the downstream firms will react to its licensing offers in the following stages. Note that due to our assumption of symmetry of demand functions, the monopoly royalty rates r_1^m and r_2^m, which maximize $r_1q_1(r_1, r_2) + r_2q_2(r_1, r_2)$, are symmetric and equal to the monopoly royalty rate r^m in the baseline model.

If the patent is weak, firm P cannot credibly threaten to go to court, even in the best case where it sets the monopoly royalty rates $r_1 = r_2 = r^m$. Therefore, both firms refuse the license, there is no litigation and firm P makes no profit. As in the baseline model, this happens when $\theta \leq \hat{\theta}_W$. At the other extreme, if the patent is strong, both firms accept the license at the monopoly royalty rates $r_1 = r_2 = r^m$. In this case, firm P makes monopoly royalty profits. As in the baseline model, this happens when $\theta \geq \hat{\theta}_S$.

Finally, consider patents of intermediate strength, with $\theta \in (\hat{\theta}_W, \hat{\theta}_S)$. First, if the litigation condition (LC_S^D) holds, firm P can set the maximum royalty rates such that both firms accept the license, i.e., $\pi_i(r_1, r_2) \geq \pi(0, 0) - L/(1-\theta)$ for $i = 1, 2$. Note that both constraints are necessarily binding. Indeed, assume that one constraint is not binding, say for firm 2. Then, firm P could increase r_2, earning higher royalties from firm 2, and this would also relax the participation constraint for firm 1. Thus, the two participation constraints are binding. Due to the symmetry of our problem, the maximum royalty rates for firms 1 and 2 are then equal, and they are equal to the royalty rate such that condition (PC_B) binds in the baseline model. As in the baseline model, firm P can charge this limit royalty rate as long as (LC_S) and (PC_B) can hold simultaneously, that is, if $\theta \in [\hat{\theta}_2, \hat{\theta}_S]$. For this range of patent strength, the patent holder can alternatively set the monopoly royalty rates $r_1 = r_2 = r^m$ and go to court.

Second, if (LC_S^D) and (PC_B^D) cannot hold simultaneously, firm P can license at most one firm, say firm 2. In this case, firm P sets the royalty rates r_1 and r_2 to maximize its royalties from firm 2, subject to the constraints that (PC_S^D) holds and $\theta(r_1 q_1 + r_2 q_2) \geq L$. Given that the outcome of the licensing game is asymmetric, firm P may then set different royalty rates for the two firms. For instance, it is the case in the Cournot example. If this optimisation problem has a solution, firm P either sells a license to firm 2 at the limit royalty rate, or sets the monopoly royalty rate and go to court. If there is no solution to the optimisation problem (i.e., there is no royalty rate accepted by firm 2 and which generates a credible litigation threat), the only option for the patent holder is to set the monopoly royalty rate and go to court.

Appendix H: Injunction

The timing of the game is the same as for the baseline model, except for the last stage, where we introduce the possibility of an injunction. So, in the first stage, firm P sets its per-unit royalty rate, r, and offers a license to firm 1 and firm 2. In the second stage, the downstream firms decide simultaneously and non-cooperatively whether to accept the license. In the third stage, firm P can finally litigate the firms that have refused the license. An injunction is issued against the infringing firms if the patent is upheld.

Stage 3: Firm P's litigation decision.

At Stage 3, firm P can litigate the infringing firms. First, consider the case where no firm has accepted the license. If firm P goes to court and the patent is upheld, it can obtain an injunction order against the two infringing firms. In this case, firm P renegotiates the licensing terms with both firms and asks the monopoly royalty rate r^m. Thus, firm P decides to go to court if and only if $2\theta r^m q(r^m, r^m) \geq L$ (i.e., if condition (LC_B^I) holds), which is equivalent to $\theta \geq \hat{\theta}_W$. In the Cournot model, condition (LC_B^I) is equivalent to $\theta \geq 6L$. We can then define the threshold royalty rate $r_L^{B,I}(\theta)$ such that firm P goes to court iff $r \geq r_L^{B,I}(\theta)$, by setting $r_L^{B,I}(\theta) = 1$ if $\theta < 6L$ and $r_L^{B,I}(\theta) = 0$ if $\theta \geq 6L$. Note that $r_L^{B,I}(\theta) \leq r_L^B(\theta)$.

Second, consider the case where one firm has accepted the license, but not the other. If firm P sues the infringing firm and the patent is upheld, it can renegotiate its licensing terms with this firm and ask the royalty rate $\hat{r}(r) = \arg\max_{r'} rq_1(r, r') + r'q_2(r, r')$. Thus, firm P sues the infringing firm if and only if $\theta\,[rq_1(r, \hat{r}) + \hat{r}q_2(r, \hat{r})] \geq L + rq_1(r, 0)$, that is, if condition (LC_S^I) holds. In the Cournot model, we find that $\hat{r}(r) = (1 + 2r)/4 \in [r, r^m]$ and that P goes to court iff $\theta/24 + r\,(3\theta - 2 + r(4 - 3\theta))\,/6 \geq L$. The set of values of r that satisfy this inequality is larger than the set of values of r such that $r \geq r_L^S(\theta)$.

Note that condition (LC_S^I) implies condition (LC_B^I). Indeed, when both firms infringe rather than only one, firm P expects a larger revenue from going to court due to better licensing terms, and it is also less costly to litigate due to the absence of an opportunity cost.

Stage 2: Downstream firms' licensing decisions.

At Stage 2, the two downstream firms decide simultaneously and non-cooperatively whether to accept the license, anticipating firm P's litigation decision at the next stage.

First, consider the case where (LC_B^I) does not hold, which implies that (LC_S^I) does not hold either. Since litigation is not a credible threat, it is a dominant strategy for each firm to refuse the license.

Second, assume that (LC_B^I) holds, but not (LC_S^I). Consider then firm i's decision to accept the license, given the decision taken by firm $j \neq i$. If firm j accepts the license, there is no risk for firm i of being brought to court when refusing the license. So, its best response is to refuse it.

If firm j refuses the license, firm i compares its expected profits if it accepts the license and if it refuses it and firm P goes to court. If firm i accepts the license and pays royalties, it will compete at a cost disadvantage against firm j, which will not be sued for infringing the patent. Firm i's profit in this case is $\pi_1(r, 0)$. If firm i refuses the license, firm P will go to court against both firms. If the patent is upheld, they will have to pay the renegotiated monopoly royalty rate r^m. Firm i's expected profit in this case is $\theta\pi(r^m, r^m) + (1 - \theta)\pi(0, 0) - L$. Therefore, firm i accepts the license if and only if $\pi_1(r, 0) \geq \theta\pi(r^m, r^m) + (1 - \theta)\pi(0, 0) - L$, that is, if condition (PC_S^I) holds. In the Cournot model, this condition is equivalent to $r \leq r_P^{S,I}(\theta) \equiv \frac{1}{2} - \frac{1}{4}\sqrt{4 - 36L - 3\theta}$, and we have $r_P^{S,I}(\theta) \geq r_P^S(\theta)$.

Finally, assume that both (LC_B^I) and (LC_S^I) hold. Consider first that firm j accepts the license. If firm i follows suit, both firms will pay royalties and obtain the same profit, $\pi(r, r)$. If firm i refuses the license, firm P will go to court. If the patent is upheld, the court orders an injunction, in which case firm 2 has to pay the renegotiated royalty rate $\hat{r}(r)$. Therefore, firm 2 accepts the license if and only if $\pi(r, r) \geq \theta\pi_2(r, \hat{r}) + (1 - \theta)\pi(0, 0) - L$, that is, if condition (PC_B^I) holds. In the Cournot model, this condition is equivalent to $r \leq r_P^{B,I}(\theta) \equiv 1 - \frac{1}{2}\sqrt{4 - 36L - 3\theta} = 2r_P^{S,I}(\theta)$, and we have $r_P^{B,I}(\theta) \geq r_P^B(\theta)$.

Second, consider that firm j refuses the license. If firm i accepts the license, firm P will litigate firm j. In this case, firm i's expected profit is $\theta\pi_1(r, \hat{r}(r)) + (1 - \theta)\pi(0, 0)$. If firm i

refuses the license, firm P will sue both firms and firm i's expected profit is $\theta\pi_1(r^m, r^m) +$ $(1-\theta)\pi(0,0) - L$. We make the (reasonable) assumption that $\pi_1(r, \hat{r}(r)) \geq \theta\pi(r^m, r^m)$. For instance, this is true if we assume that $\partial\pi(r,r)/\partial r \leq 0$ and $\partial\pi_1(r_1, r_2)/\partial r_2 \geq 0$, two conditions that hold for the Cournot model with homogeneous products and general demands and for the model of differentiated Bertrand competition (see Amir et al. (2014)). Under the assumption that $\pi_1(r, \hat{r}(r)) \geq \theta\pi_1(r^m, r^m)$, firm i's best response is to accept the license if firm j refuses it.

We can now characterize the equilibria of licensing game for a given royalty rate r.

If (LC_B^I) does not hold, it is a dominant strategy for each firm to refuse the license. Therefore, the only equilibrium if (R, R) and firm P does not go to court.

If (LC_B^I) holds, but not (LC_S^I), firm i accepts the license if firm j refuses it and (PC_S^I) holds; otherwise, firm i refuses the license. Therefore, if (PC_S^I) holds, there are two equilibria, (A, R) and (R, A), and firm P does not litigate the infringer. If (PC_S^I) does not hold, the only equilibrium if (R, R) and firm P goes to court.

If (LC_B^I) and (LC_S^I) hold, firm i accepts the license if firm j refuses it or if firm j accepts it and (PC_B^I) holds; otherwise, firm i refuses the license. Therefore, if (PC_B^I) holds, the only equilibrium is (A, A). If if (PC_B^I) does not hold, there are two equilibria, (A, R) and (R, A), and firm P litigates the infringer.

Stage 1: Firm P's decision. Define $\hat{\theta}^I$ as the intersection of (LC_S^I) and (PC_B^I). Since, compared to the baseline case, (LC_S^I) can be satisfied with a lower royalty rate and (PC_B^I) be satisfied with a higher royalty rate, we have $\hat{\theta}^I < \hat{\theta}_2$. We then solve for the equilibrium royalty rate for the different relevant ranges of the patent strength θ.

(1) If $\theta < \hat{\theta}_W$, firm P's litigation threat is never credible. So, no firm is licensed and there is no litigation.

(2) If $\theta \in [\hat{\theta}_W, \hat{\theta}^I)$, there are three possible outcomes to the licensing game. First, for low values of r such that (PC_S^I) holds, one firm takes a license, the other refuses it, and there is no litigation. For larger (intermediate) values of r, condition (PC_S^I) does not hold. Therefore, the two firms refuse the license and firm P goes to court. Finally, for larger values of r, condition (LC_S^I) holds but not (PC_S^I). Therefore, one firm takes a license, the other does not, and firm P goes to court. Therefore, firm P either sets $r^\star$ such that (PC_S^I) binds and only one license is sold, or it sets $r^\star = r^m$ and goes to court.

(3) If $\theta \in [\hat{\theta}^I, \hat{\theta}_S)$, firm P can either sell a license to both firms at the royalty rate such that (PC_B^I) binds, or set $r^\star = r^m$ and go to court.

(4) If $\theta \geq \hat{\theta}_S$, the firms accept the license at the monopoly royalty rate; thus, firm P sets $r^\star = r^m$ and there is no litigation.