The Journal of Industrial Economics

WILEY

ORIGINAL ARTICLE

Interoperability Between Ad-Financed Platforms With Endogenous Multi-Homing

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Accepted: 5 January 2026

Keywords: advertising | interoperability | multi-homing | platform competition

ABSTRACT

Platform interoperability is considered a powerful tool to promote competition in digital markets when network effects are at play. We study the incentives of two competing ad-funded platforms to provide interoperability in a setting where consumers can single-home or multi-home and decide how much time to spend online. When the platforms are symmetric, perfect interoperability emerges in equilibrium and is socially efficient. When a larger platform has an installed base of customers, its incentive to make its services interoperable is lower than for the smaller platform. Interoperability does not emerge in equilibrium if the installed base is sufficiently large. However, mandating interoperability between the asymmetric platforms is not always socially optimal.

JEL Classification: C12, C22, G01

> "The Internet was made in universities, and it was designed to interoperate. And as we've commercialized it, we've added more of an island-like approach to it, which I think is somewhat a shame for users."
> Larry Page, Google cofounder, December 11, 2012.[1]

1 | Introduction

The Internet was originally designed as an interoperable network between open systems. For example, email services are based on an open and interoperable protocol for online communication, regardless of a person's email service or the type of device used to send emails. In contrast, today's most widely used instant messaging services rely on proprietary protocols, and users cannot send messages from one messaging service to another (e.g., from WhatsApp to Signal or Snapchat). Thus, consumers have no choice but to join the same platform as the users they want to communicate with. As a result, network effects play a key role in the competition between messaging service providers.

However, strong network effects can make it difficult for new entrants to compete with incumbent platforms. Users are often reluctant to switch to a new platform, even if it offers better services, because they would have to coordinate their move and may face individual switching costs, such as rebuilding their profile or providing data again. While platform differentiation and multi-homing may allow multiple platforms to coexist in the market, a platform with a large installed base will still be more attractive to consumers than a smaller one.

Recently, policymakers and academics (see, e.g., [1–10]) have been advocating for greater interoperability between digital platforms. In September 2022, the European Commission (EC) introduced the Digital Market Act (DMA),[2] a regulatory framework aimed at promoting fairness and contestability in digital markets. In particular, the new law mandates large dominant platforms (the so-called "gatekeepers") offering messaging services to provide interoperability with smaller messaging platforms upon request and free of charge. The EC will assess the extension of this interoperability provision to social media in the future. Similar legislation, the ACCESS Act, is currently under consideration in the U.S. Congress.[3]

In this paper, we study the incentives of ad-financed messaging platforms to make some features of their services interoperable, and how these incentives compare to the social optimum. We develop a model in which two horizontally differentiated platforms offer communication services to consumers. Consistent with the business model of messaging service providers, we assume that the platforms are free to users and generate revenue solely through advertising. Consumers can decide to single-home on a given platform or to multi-home, taking into account the size of each network. Once they join a platform, consumers decide how much time they want to spend communicating with other users, considering the nuisance from advertising on the platform. A consumer's exposure to advertising—and the nuisance it causes—increases with the amount of time spent on the platform.

We first consider a benchmark case where the two platforms are symmetric. We show that interoperability reduces multi-homing on the consumer side, decreasing the total demand addressed to each platform. The composition of demand is also affected, with an increase in single-homing and a decrease in multi-homing for each platform.

Interoperability has conflicting effects on platform profits. First, greater interoperability enhances the market power of the platforms relative to advertisers (the *market power* effect). Interoperability increases the single-homing demand of each platform, which benefits the platforms as they can charge higher ad prices for (exclusive) single-homers than for (non-exclusive) multi-homers. This is because even though multi-homers spend more time communicating than single-homers and are thus more exposed to advertising, a platform can only charge advertisers for the incremental value it provides. Second, a higher level of interoperability reduces the total demand of each platform, which shrinks the audience they offer to advertisers and reduces their profits (the *total viewership* effect). Third, as interoperability increases, single-homers spend more time communicating on their host platform. This increases their exposure to advertising, leading to higher ad revenues for the platforms (the *usage intensification* effect).

Overall, the (positive) market power and usage intensification effects always dominate the (negative) total viewership effect. Therefore, platforms have a strong incentive to interoperate, and without any cost of implementing interoperability, perfect interoperability emerges in equilibrium.

We show that the equilibrium outcome is efficient from a social point of view. This is because consumers also prefer perfect interoperability, in particular because it increases the quality of interactions between the single-homers of each platform, while advertisers are indifferent because their surplus is always fully extracted by the platforms. Therefore, in a symmetric environment, the level of interoperability chosen by the platforms should not be a concern. This result becomes more nuanced when we consider the fixed costs of implementing interoperability. In this case, the equilibrium level of interoperability may be too low compared to the social optimum when implementation costs are intermediate. This is because, when choosing its level of interoperability, a platform does not consider the positive effect of interoperability on consumer surplus.

However, the market for messaging services is highly asymmetric, with a few dominant players (e.g., Facebook Messenger) facing smaller rivals (e.g., Telegram). Therefore, we next consider the case where one platform is larger than its competitor. Specifically, we assume that one platform has an installed base of users. We then examine the incentives of the large and small platforms to make their services interoperable, and how the asymmetry between them affects these incentives.

We find that the large platform can charge higher ad prices for single-homers and multi-homers than the small platform, and that the difference in ad prices increases with the size of the installed base. However, greater interoperability reduces this price differential, thereby leveling the playing field between the small and large platforms in the advertising market. Similarly, the large platform's installed base advantage allows it to attract more users than the small platform. Again, greater interoperability mitigates this advantage by reducing the difference in total consumer demand between the small and large platforms, thereby leveling the playing field in terms of network effects.

In this asymmetric environment, we find that the large platform is less willing to interoperate than the small platform. This is because the (positive) market power and usage intensification effects of interoperability on profits are stronger for the small platform than for the large platform, while the (negative) total viewership is smaller in absolute terms. As the level of interoperability is determined by the preferences of the platform that values it the least, the equilibrium level of interoperability is determined by the largest platform. We find that if the installed base advantage is large enough, interoperability does not arise in equilibrium.

The equilibrium level of interoperability is weakly too low for consumers and total welfare. Therefore, in some cases, interoperability may not materialize even though it would be efficient for consumers and the welfare. However, the absence of interoperability in equilibrium does not necessarily justify regulatory intervention. Although interoperability levels the playing field between platforms, it may reduce consumer surplus and total surplus in highly asymmetric markets. Specifically, we find that mandating interoperability can harm consumers and welfare when the dominant platform's installed base is very large.

1.1 | Related Literature

We contribute to the literature analyzing firms' incentives to make their products compatible (interoperable) in the presence of network effects (see, e.g., Katz and Shapiro [11] and Farrell and Klemperer [12]). Whether compatibility arises in equilibrium depends on the balance between two opposing forces: the demand expansion effect and the leveling effect. On the one hand, compatibility increases the value of firms' products by enhancing network benefits. As they can attract more users to the market, firms have a mutual interest in making their products compatible. On the other hand, when firms have different installed bases (see, e.g., [13–15]), compatibility diminishes the competitive advantage of larger firms, while smaller firms favor compatibility as it allows them to catch up with their larger rivals. Therefore, interoperability is likely to emerge voluntarily when firms are relatively symmetric. But when firms are asymmetric, dominant (large) firms can resist interoperability, making it less likely to emerge.

Our contribution to this literature is twofold. First, the literature either ignores multi-homing or models it exogenously. Doganoglu and Wright [16] examine firms' incentives to make their networks compatible in a setting where multi-homing can occur but is mostly exogenous.[4] Crémer, Rey and Tirole [14] consider endogenous multi-homing in an extension of their main setting, but only in the extreme case where networks are incompatible. In contrast, we fully endogenize consumers' multi-homing decisions and show that interoperability reduces multi-homing, which affects platforms' incentives to interoperate. Second, while the literature focuses on subscription-based business models, we consider advertising-based business models (i.e., the type of business model used by messaging service providers or social media). Our analysis reveals three novel effects of interoperability on profits in this context: the *market power*, *total viewership*, and *usage intensification* effects. The *market power* and *total viewership* effects arise because interoperability affects multi-homing, and multi-homers are less valuable to advertisers than single-homers. The *usage intensification* effect arises because greater interoperability increases the time that single-homers spend online and thus their exposure to advertising.

Our paper is also closely related to Bourreau and Kraemer [17]. Like in the present paper, they show that interoperability reduces multi-homing and that this can have important implications for competition between digital platforms. However, their focus is different; they examine the impact of mandating interoperability on market contestability, that is, the possibility for a more efficient entrant to displace an incumbent platform, when the market has a strong tendency to tip. By contrast, we consider a setting in which two platforms can both be active in the market and study whether interoperability can emerge as the outcome of non-cooperative strategies depending on the degree of asymmetry between them. Besides, while Bourreau and Kraemer [17] propose a dynamic model of network adoption with consumer heterogeneity in network effects and a fixed ad revenue per capita, in the present paper, we model the amount of time that consumers spend online and analyze the impact of interoperability on the advertising market.

We also relate to the literature on ad-financed media markets in the presence of shared viewership (see, e.g., [18–21]). This literature emphasizes the importance of demand composition as opposed to total demand, given that firms compete on the advertiser side only for multi-homing users while acting as gatekeepers for single-homers. In particular, Anderson, Foros and Kind [20] show that the *incremental pricing principle*, according to which multi-homing consumers are less valuable than single-homing consumers, has implications for firms' revenues and their incentives to differentiate. The idea is that if firms earn more profit from their single-homing users, they will avoid differentiation to minimize multi-homing.

We contribute to this literature by developing a model in which exposure to advertising depends on the amount of time consumers spend online. Relatedly, Anderson and Peitz [22] study advertising congestion in a model where a representative consumer allocates a fixed amount of time across media.[5] Our modeling approach differs in two key ways: we allow the amount of time that consumers spend online to be endogenous, and we consider heterogeneous consumers, resulting in an equilibrium where some consumers single-home while others multi-home.

1.2 | Structure of the Paper

The rest of the paper is organized as follows. In the next section, we present the model. In Section 3, we solve for the equilibrium level of interoperability between two symmetric platforms and compare it to the social optimum. In Section 4, we extend our framework to consider interoperability between a small and a large platform. We conclude in Section 5.

2 | Model

We study a model in which two communication platforms (e.g., messaging platforms) compete for consumers and advertisers, and decide non-cooperatively on their level of interoperability.

2.1 | Platforms

Two platforms, A and B, offer communication services to consumers. In line with prevailing industry practice for messaging services, we assume that the platforms are free to users and generate revenue through advertising.[6] Specifically, each platform $i \in \{A, B\}$ charges advertisers a price per ad p_i, but users can access the platforms for free. However, as we will see, the presence of ads on the platforms is a nuisance to them.[7] The platforms incur no costs in providing services to consumers and advertisers.

The platforms offer a mass of communication features, such as sending and receiving text or video messages, or sharing a location. Each platform i chooses a subset of features $[0, \phi_i]$ to make interoperable with the other platform, with $\phi_i \in [0, 1]$. Communication is only effective if the sending and receiving parties use the same features. So, the resulting level of interoperability ϕ between platforms is determined by their common features, that is, $\phi = \min\{\phi_A, \phi_B\}$. Thus, a consumer using platform i can interact on-net with other users of platform i with the full set of features (i.e., a quality of 1), and off-net with users of the other platform through interoperability, but with a lower interaction quality $\phi \leq 1$. We assume that implementing interoperability is costless for the platforms.[8]

2.2 | Consumers

The platforms are horizontally differentiated à la Hotelling, with platform A located at $y_A = 0$ and platform B at $y_B = 1$ on the unit interval. A unit mass of consumers is uniformly distributed over this interval. Each consumer can decide to single-home (join one platform) or multi-home (join both).

A consumer located at y who single-homes on platform $i \in \{A, B\}$ derives the utility

$$v_0 - t|y - y_i| + u(\tau_i)(n_i + n_{i,j}) + \phi u(\hat{\tau}_i)n_j,$$

where v_0 is the stand-alone benefit of joining the platform,[9] $t|y - y_i|$ represents the transportation cost (e.g., the utility loss due to preference mismatch), $u(\tau_i)(n_i + n_{i,j})$ is the gross surplus from on-net interactions with the n_i single-homing users of platform i and the $n_{i,j}$ multi-homers, and $\phi u(\hat{\tau}_i)n_j$ is the gross surplus from off-net interactions with the n_j single-homing users of platform $j \neq i$.[10] The gross surplus of an on-net interaction is given by

$$u(\tau) = \kappa \frac{\tau^{1-\frac{1}{\beta}}}{1 - \frac{1}{\beta}},$$

where τ is the time spent communicating and $\beta > 1$ is a measure of demand elasticity.[11] The gross surplus of an off-net interaction is $\phi u(\hat{\tau})$, reflecting the lower quality of interoperable communications ($\phi \leq 1$).

When using a platform to communicate, consumers are exposed to advertising. Specifically, we assume that the more time a user spends on the platform, the more advertising she is exposed to, which is perceived as a nuisance. Formally, if the user spends an amount of time τ_i on platform i, the disutility from advertising is $\gamma r_i^e \tau_i$, where $\gamma > 0$ and r_i^e denotes the ad level (i.e., the number of advertisers) expected on platform i. Thus, the net utility of an on-net communication is $u(\tau_i) - \gamma r_i^e \tau_i$. Similarly, the net utility of an off-net (interoperable) communication of duration $\hat{\tau}_i$ is $\phi u(\hat{\tau}_i) - \gamma r_i^e \hat{\tau}_i$.

Finally, a multi-homing consumer receives the stand-alone benefit $(1 + \eta)v_0$ by joining both platforms, where $\eta \leq 1$ captures the overlap in the standalone benefits offered by each platform. Multi-homing consumers always use the best available technology to interact with another user. They can choose to communicate with a single-homing user either on-net, using the same platform as the user, or off-net using the other platform via interoperability. Multi-homers can interact with other multi-homers twice, but only one interaction has value (i.e., there is no double-counting of network benefits).[12] Thus, multi-homers derive the utility

$$(1 + \eta)v_0 - t + \sum_{\substack{i=A,B \\ j \neq i}} \max\{u(\tau_i) - \gamma r_i^e \tau_i, \phi u(\hat{\tau}_j) - \gamma r_j^e \hat{\tau}_j\}n_i$$
$$+ \max\{u(\tau_A) - \gamma r_A^e \tau_A, u(\tau_B) - \gamma r_B^e \tau_B\}n_{A,B}.$$

2.3 | Advertisers

A unit mass of homogeneous advertisers wants to inform consumers about their products. We assume that a fraction μ of consumers are interested in each advertised product and have a willingness to pay ω, while the rest have a willingness to pay of 0. Advertisers are monopolists in their product markets, and thus charge the price ω.

Each advertiser decides on which platform(s) to advertise on, with at most one ad per platform. The probability ϵ that a consumer will pay attention to an ad increases with the amount of time τ of exposure to the ad across all platforms, so $\epsilon = \epsilon(\tau)$, with $\epsilon'(\tau) > 0$. Therefore, the expected value of this consumer to an advertiser is $\pi^e = \mu\omega\epsilon(\tau)$. For simplicity, we assume that $\epsilon(\tau)$ is linear, that is, $\epsilon(\tau) = \epsilon_0\tau$. We assume that $\kappa < \gamma$ to ensure that $\tau \leq 1$ in equilibrium and that ϵ_0 is small enough so that we always have $\epsilon(\tau) \in [0, 1]$. Then, the expected value of the consumer to the advertiser can be written as $\pi^e = \sigma\tau$, with $\sigma \equiv \epsilon_0\mu\omega$.

2.4 | Timing

First, platforms non-cooperatively choose their level of interoperability ϕ_i. The resulting level of interoperability $\phi = \min\{\phi_A, \phi_B\}$ is made public. Second, platforms simultaneously choose their price per ad p_i. Advertisers observe the ad prices and decide on which platform(s) to buy ad space, anticipating the number of users on each platform. Third, consumers decide which platform(s) to join and how much time they want to spend communicating on-net and off-net. Following Anderson et al. [20], we assume that consumers do not observe the ad prices or the ad levels, but rationally anticipate them.

We look for the Subgame-Perfect Nash Equilibrium of this game. Consumers form rational expectations about the user base of each platform. The fact that consumers do not observe the ad prices or the ad levels means that they do not respond instantaneously to a change in ad levels, which seems reasonable in the context of messaging services. Following Crémer et al. [14], we also assume that the equilibrium level of interoperability is $\min\{\phi_A^\star, \phi_B^\star\}$, where $\phi_i^\star$ is the level of interoperability that maximizes platform i's profit.[13]

In the next section, we solve for the equilibrium of the game with symmetric platforms. In Section 4, we introduce platform asymmetry by considering that one platform initially has an installed base of users. All proofs can be found in the Appendix.

3 | Symmetric Platforms

In this section, we solve for the equilibrium when platforms are symmetric. As usual, we solve the game backwards, starting from the last stage.

3.1 | Choice of Time Spent Communicating

In the last stage, consumers decide how much time to spend communicating with other users.

Consider a consumer who single-homes on platform $i = A, B$. The consumer chooses the amount of time τ spent communicating on-net with other users to maximize her net surplus, $u(\tau) - \gamma r_i^e \tau$. Solving for the first-order condition,[14] we obtain the optimal time spent communicating on-net, $\tau^*(r_i^e) = \left(\frac{\gamma r_i^e}{\kappa}\right)^{-\beta}$, which decreases with the expected ad level r_i^e. Thus, when users are exposed to more (annoying) ads, they spend less time communicating on the platform, with β representing the (constant) elasticity of platform usage to the level of advertising. The associated surplus derived from an on-net communication is

$$\alpha(r_i^e) = u(\tau^*(r_i^e)) - \gamma r_i \tau^*(r_i^e) = \frac{1}{\beta - 1}\frac{(\gamma r_i^e)^{-(\beta-1)}}{\kappa^{-\beta}},$$

which decreases with the expected level of advertising as $\alpha'(r_i^e) = -\gamma\tau^*(r_i^e) < 0$.

Similarly, the consumer chooses the amount of time $\hat{\tau}$ spent communicating off-net with other users to maximize her net surplus, $\phi u(\hat{\tau}) - \gamma r_i^e \hat{\tau}$. Solving for the first-order condition gives the optimal amount of time spent communicating off-net, $\hat{\tau}^*(r_i^e) = \left(\frac{\gamma r_i^e}{\phi\kappa}\right)^{-\beta}$. Note that for a given level of advertising, consumers spend less time communicating off-net than on-net ($\hat{\tau}^*(r_i^e) \leq \tau^*(r_i^e)$) due to imperfect interoperability ($\phi \leq 1$). The associated net surplus from an off-net communication is

$$\hat{\alpha}(r_i^e) = \phi u(\hat{\tau}^*(r_i^e)) - \gamma r_i^e \hat{\tau}^*(r_i^e) = \frac{1}{\beta - 1}\frac{(\gamma r_i^e)^{-(\beta-1)}}{(\phi\kappa)^{-\beta}},$$

which decreases with the expected level of advertising r_i^e.

Now, we can define $\theta \equiv \hat{\alpha}(r)/\alpha(r) = \phi^\beta$, so that $\hat{\tau}^*(r) = \theta\tau^*(r)$ and $\hat{\alpha}(r) = \theta\alpha(r)$. Note that $\theta \in [0,1]$, as $\phi \in [0,1]$. The utility of a consumer single-homing on platform $i = A, B$ (gross of transportation costs) can then be written in a simple way as:

$$v_0 + \alpha(r_i^e)\left[n_i + n_{i,j} + \theta n_j\right], \tag{1}$$

where n_i and n_j are the number of single-homing users of platform i and platform $j \neq i$, respectively, and $n_{i,j}$ is the number of multi-homers.

In the utility function (1), θ represents the effective level of interoperability for consumers. It increases with the technical level of interoperability ϕ and decreases with the elasticity of demand β. In the rest of the analysis, we will refer to θ as the level of interoperability and consider that each platform i chooses a unilateral level of interoperability θ_i instead of ϕ_i.

3.2 | Consumer Homing Decisions

We now turn to the consumer's decision on which platform(s) to join. First, we determine which platform the multi-homers use to interact with other users. Then, we analyze the consumer's choice between single-homing and multi-homing. In this analysis, we assume that $r_A^e \leq r_B^e$, which is without loss of generality since the two platforms are symmetric.

3.2.1 | Choice of Communication Platform for Multi-Homers

A multi-homing consumer always uses the best available technology to communicate with other users, choosing the platform that delivers the highest surplus from interactions.

Lemma 1 (Platform choice for multi-homers). *If $\alpha(r_B^e) \geq \theta\alpha(r_A^e)$, a multi-homer will use platform B to communicate with platform B's single-homers, and platform A to communicate with all other users. Otherwise, the multi-homer will always use platform A to communicate with other users.*

The condition $\alpha(r_B^e) \geq \theta\alpha(r_A^e)$ in the lemma means that multi-homers prefer to use platform B to communicate with the single-homers of platform B. It holds in particular when $r_A^e = r_B^e$ (as $\theta \leq 1$), so it will hold in the equilibrium of the advertising market where $r_A^e = r_B^e = 1$, as we will show below. To simplify the exposition, we thus focus on this case in the following. We will reconsider the other case where $\alpha(r_B^e) < \theta\alpha(r_A^e)$ when we determine the equilibrium in the advertising market.

3.2.2 | Single-Homing Versus Multi-Homing

We can now analyze the consumer's choice between single-homing and multi-homing.

Consumers form rational expectations about platforms' user bases, expecting x_A^e users on platform A and $1 - x_B^e$ users on platform B. Based on the above analysis, the expected net utility of multi-homing is:

$$u_{AB} = (1+\eta)v_0 - t + \alpha(r_A^e)x_A^e + \alpha(r_B^e)(1 - x_A^e). \tag{2}$$

The expected net utility of a consumer located at x and single-homing on A is given by:

$$u_A = v_0 + \alpha(r_A^e)\left[x_A^e + \theta(1 - x_A^e)\right] - tx, \tag{3}$$

which is positive if and only if

$$x \leq \overline{x}_A \equiv \frac{v_0 + \alpha(r_A^e)\left[x_A^e + \theta(1 - x_A^e)\right]}{t},$$

and the expected net utility of single-homing on B is

$$u_B = v_0 + \alpha(r_B^e)\left[1 - x_B^e + \theta x_B^e\right] - t(1 - x). \tag{4}$$

Using (2) and (4), we can compute the incremental utility the consumer gains by joining platform A in addition to platform B:

$$u_{AB} - u_B = \eta v_0 - tx + [\alpha(r_A^e) - \alpha(r_B^e)][x_A^e - x_B^e] + [\alpha(r_A^e) - \theta\alpha(r_B^e)]x_B^e.$$

By joining platform A in addition to platform B, the consumer gains additional standalone benefits of ηv_0, but incurs additional transportation costs tx. Additionally, the consumer can interact with other multi-homers on platform A rather than on platform B, which provides an additional interaction benefit $\alpha(r_A^e) - \alpha(r_B^e)$ since $r_A^e \leq r_B^e$. Furthermore, the consumer can interact on-net with platform A's single-homers rather than off-net through platform B via interoperability, which brings an additional interaction benefit $\alpha(r_A^e) - \theta\alpha(r_B^e)$.

Therefore, consumers of type $x \leq \hat{x}_A$ join platform A in addition to platform B, with

$$\hat{x}_A = \frac{\eta v_0 + \left[\alpha(r_A^e) - \alpha(r_B^e)\right]x_A^e + (1-\theta)\alpha(r_B^e)x_B^e}{t}. \tag{5}$$

Assuming that consumers' expectations about network sizes imply some degree of multi-homing (i.e., $x^e_A > x^e_B$), we have $\hat{x}_A < \overline{x}_A$. Therefore, consumers of type $x \leq \hat{x}_A$ join platform A either as single-homers or as multi-homers.

Similarly, using (2) and (3), the incremental gain from joining platform B in addition to platform A is:

$$u_{BA} - u_A = \eta v_0 - t(1-x) + \left[\alpha(r^e_B) - \theta\alpha(r^e_A)\right](1 - x^e_A),$$

where $u_{BA} = u_{AB}$. Thus, users of type $x \geq \hat{x}_B$ join platform B in addition to platform A, with

$$\hat{x}_B = 1 - \frac{\eta v_0 + \left[\alpha(r^e_B) - \theta\alpha(r^e_A)\right](1 - x^e_A)}{t}. \tag{6}$$

We have $\partial\hat{x}_A/\partial x^e_B < 1$ if and only if $t > (1-\theta)\alpha(r^e_B)$ and $\partial\hat{x}_B/\partial x^e_A < 1$ if and only if $t > \alpha(r^e_B) - \theta\alpha(r^e_A)$. As we will see below, we have $r^e_A = r^e_B = 1$ in equilibrium. Therefore, to ensure the existence of a stable equilibrium where both platforms are active, we must assume that $t > (1-\theta)\alpha(1)$ for all interoperability levels $\theta \in [0,1]$, which is true with the following assumption:

Assumption 1 (Market sharing). $t > \alpha(1)$.

In equilibrium, we have $x^e_i = \hat{x}_i$ for $i = A, B$. Solving for the system of equations defined by (5) and (6), we find the marginal consumers indifferent between single-homing and multi-homing, $\hat{x}_A$ and $\hat{x}_B$. Define $\lambda(r^e_A, r^e_B) \equiv \alpha(r^e_A) - \alpha(r^e_B)$ and $\psi(r^e_A, r^e_B) \equiv \alpha(r^e_B) - \theta\alpha(r^e_A)$. We have:

$$\hat{x}_A(r^e_A, r^e_B) = \frac{\eta v_0(t - (1-\theta)\alpha(r^e_B)) + (1-\theta)\alpha(r^e_B)(t - \psi(r^e_A, r^e_B))}{t(t - \lambda(r^e_A, r^e_B)) - (1-\theta)\alpha(r^e_B)\psi(r^e_A, r^e_B)},$$

$$\hat{x}_B(r^e_A, r^e_B) = \frac{(t - \lambda(r^e_A, r^e_B) - \eta v_0)(t - \psi(r^e_A, r^e_B)) + \eta v_0\lambda(r^e_A, r^e_B)}{t(t - \lambda(r^e_A, r^e_B)) - (1-\theta)\alpha(r^e_B)\psi(r^e_A, r^e_B)}.$$

Figure 1 shows the demand structure when there is partial multi-homing ($0 < \hat{x}_B < \hat{x}_A < 1$). Consumers located at $x < \hat{x}_B$ single-home on platform A, while consumers of type $x > \hat{x}_A$ single-home on platform B. Consumers located between $\hat{x}_B$ and $\hat{x}_A$ multi-home. The number of single-homers on platforms A and B are then $SH_A(r^e_A, r^e_B) \equiv \hat{x}_B(r^e_A, r^e_B)$ and $SH_B(r^e_A, r^e_B) \equiv 1 - \hat{x}_A(r^e_A, r^e_B)$, respectively, and there are $MH(r^e_A, r^e_B) \equiv \hat{x}_A(r^e_A, r^e_B) - \hat{x}_A(r^e_A, r^e_B)$ multi-homers.

Note that, as we have assumed that consumers do not observe ad prices or ad levels, the numbers of single-homers and multi-homers depend on consumers' expectations about ad levels (r^e_i), rather than on the realized ad levels.

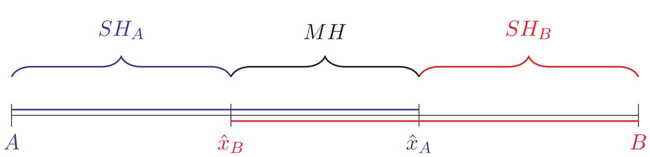

FIGURE 1 | Demand structure with partial multi-homing. *Notes:* [Colour figure can be viewed at wileyonlinelibrary.com]

3.3 | Advertising Decisions

In Stage 2, the platforms set their advertising price p_i, and then advertisers decide on which platform(s) to buy advertising space.

To simplify the exposition, we denote $\alpha \equiv \alpha(1)$ and $\tau^* \equiv \tau^*(1)$ (with a little abuse of notation). The following lemma characterizes the equilibrium in the advertising market.[15]

Lemma 2 (Incremental pricing with usage-based advertising). *There is a unique equilibrium in which each platform $i \in \{A, B\}$ sets the advertising price $p_i = \sigma\tau^{SH} SH_i(1,1) + \sigma\tau^* MH(1,1)/2$, where $\tau^{SH} = \left(\hat{x}_A(1,1) + \theta(1 - \hat{x}_A(1,1))\right)\tau^*$ is the time spent online by single-homers, and the numbers of single-homers and multi-homers are*

$$SH_A(1,1) = SH_B(1,1) = \frac{t - \eta v_0}{t + (1-\theta)\alpha} \quad \text{and}$$
$$MH(1,1) = \frac{2\eta v_0 + (1-\theta)\alpha - t}{t + (1-\theta)\alpha}.$$

Each advertiser places an ad on each platform, and users rationally expect $r^e_i = 1$.

Therefore, as in Anderson et al. [20], each platform charges advertisers a price equal to the value of its exclusive single-homing users, plus the incremental value associated with multi-homers, and all advertisers multi-home in equilibrium.[16]

For simplicity, for the rest of the analysis, we will use the notation $\hat{x}_i$ instead of $\hat{x}_i(1,1)$, SH_i instead of $SH_i(1,1)$, and MH instead of $MH(1,1)$.

We are interested in market structures with both single-homers and multi-homers, that is, where $0 < \hat{x}_B < \hat{x}_A < 1$ in equilibrium for all $\theta \in [0,1]$. This is true if:[17]

Assumption 2 (Partial multi-homing). $t/2 < \eta v_0 < t$.

Under Assumption 2, the advertising price for a single-homer, $\sigma\tau^{SH}(\theta)$, increases with the level of interoperability θ.[18] This is because at higher levels of interoperability, single-homers spend more time on their host platform and are therefore more exposed to advertising. By contrast, the ad price for a multi-homer, $\sigma\tau^*/2$, does not depend on the level of interoperability because multi-homers never communicate via interoperability.

3.4 | Platforms' Interoperability Decisions

In Stage 1, platforms decide non-cooperatively on their level of interoperability. We begin by studying the impact of interoperability on consumer demand for each platform.

The following lemma shows that interoperability affects multi-homing.

Lemma 3 (Interoperability and multi-homing). *An increase in the level of interoperability reduces consumer multi-homing.*

To understand this result, we analyze how interoperability affects the consumer's choice between single-homing and multi-homing. Consider Equation (5), which gives the location $\hat{x}_A$ of the marginal consumer indifferent between single-homing on platform B and multi-homing. Differentiating (5) with respect to the level of interoperability θ at $r_A^e = r_B^e = 1$, we obtain

$$\frac{d\hat{x}_A}{d\theta} = \frac{\alpha}{t}\left[-\hat{x}_B + (1-\theta)\frac{d\hat{x}_B}{d\theta}\right]. \tag{7}$$

Equation (7) shows that increasing the level of interoperability has two effects on the location of the marginal consumer. First, taking the single-homing demand of platform A, $\hat{x}_B$, as given, a higher level of interoperability reduces the incentives to multi-home and pushes users to single-home on B. Indeed, for a user of platform B, a higher level of interoperability improves the quality of interactions with the exclusive users of A via interoperability, making multi-homing less attractive. A second effect is that a higher level of interoperability increases the single-homing demand of platform A (as $d\hat{x}_B/d\theta \geq 0$ under Assumption 1), making multi-homing more attractive for a given level of interoperability. Overall, we find that the first effect always dominates the second.

Lemma 3 implies that interoperability affects the demand structure of each platform in the following way:

Corollary 1 (Interoperability and consumer demand). *An increase in the level of interoperability leads to an increase in the single-homing demand of each platform and a decrease in their total demand.*

We now consider the (non-cooperative) interoperability decisions of the platforms. As in the equilibrium of Stage 2, all advertisers multi-home, platform i's profit is given by:

$$\Pi_i(\theta_i, \theta_j) = \sigma\tau^{SH}(\theta)SH_i(\theta) + \frac{\sigma\tau^*}{2}MH(\theta), \quad \text{with } \theta = \min\{\theta_i, \theta_j\}. \tag{8}$$

Single-homers are exclusive to their host platform, which can then extract all the surplus from advertisers. In contrast, multi-homers are less valuable because platforms can only charge advertisers for the incremental value. Single-homers, however, spend less time communicating than multi-homers, so their potential attention to ads is lower.

We find that in this symmetric environment, platforms have a strong incentive to interoperate:

Proposition 1 (Equilibrium level of interoperability). *The equilibrium level of interoperability is* $\theta^\star = 1$.

Although in equilibrium interoperability is perfect, interoperability has conflicting effects on the platforms' profits. To see this, we compute the effect of a higher level of interoperability on platform A's profit using Equation (8):

$$\frac{d\Pi_A}{d\theta} = \underbrace{\left(\sigma\tau^{SH}(\theta) - \frac{\sigma\tau^*}{2}\right)\frac{d\hat{x}_B}{d\theta}}_{(+)} + \underbrace{\frac{\sigma\tau^*}{2}\frac{d\hat{x}_A}{d\theta}}_{(-)} + \underbrace{\sigma\frac{d\tau^{SH}(\theta)}{d\theta}\hat{x}_B}_{(+)}. \tag{9}$$

The first term in Equation (9) represents a *market power effect*, which is positive as $d\hat{x}_B/d\theta > 0$ and $\tau^{SH}(\theta) > \tau^*/2$. When the level of interoperability increases, some multi-homers become exclusive users of platform A (the marginal consumer $\hat{x}_B$ shifts to the right) due to the substitutability between interoperability and multi-homing (Lemma 3). The platform can then charge the monopoly price $\sigma\tau^{SH}(\theta)$ instead of the incremental value $\sigma\tau^*/2$. This gain increases with the level of interoperability, since $\tau^{SH}(\theta)$ increases with θ. In other words, a higher level of interoperability increases the market power of platforms over advertisers.

The second term represents a *total viewership effect*, and it is negative since $d\hat{x}_A/d\theta < 0$. A higher level of interoperability reduces the total viewership that the platform can offer to advertisers, and therefore, reduces its profits. This is because there is less need to multi-home to interact with users on other platforms. Thus, the total user base of each platform decreases.[19]

Finally, the third term represents a *usage intensification effect*: when the level of interoperability increases, single-homers spend more time communicating on their host platform, increasing advertising revenue.

Proposition 1 shows that the two positive effects always dominate the negative effect, resulting in a maximum level of interoperability in equilibrium. Note that the usage intensification effect is key: indeed, the sum of the first two effects in Equation (9) is negative. So, if interoperability does not stimulate usage ($d\tau^{SH}/d\theta = 0$), platforms choose not to interoperate.

3.5 | Welfare Analysis

Finally, we examine the effect of interoperability on the surplus of market participants and study the optimal choice of interoperability by a welfare-maximizing regulator. We define welfare as the sum of consumer surplus and producer surplus.

Consumer surplus is given by

$$CS(\theta) = \int_0^{\hat{x}_B(\theta)} u_A(\theta, x)dx + \int_{\hat{x}_B(\theta)}^{\hat{x}_A(\theta)} u_{AB}dx + \int_{\hat{x}_A(\theta)}^1 u_B(\theta, x)dx. \tag{10}$$

Lemma 4 (Consumer surplus). *Consumer surplus increases with the level of interoperability.*

To understand this result, consider the differentiation of consumer surplus, given by (10), with respect to the level of interoperability θ:

$$\begin{aligned}\frac{dCS}{d\theta} &= \int_0^{\hat{x}_B(\theta)} \frac{\partial u_A}{\partial\theta}(\theta, x)dx + \underbrace{\int_{\hat{x}_B(\theta)}^{\hat{x}_A(\theta)} \frac{\partial u_{AB}}{\partial\theta}dx}_{=0} \\ &\quad + \int_{\hat{x}_A(\theta)}^1 \frac{\partial u_B}{\partial\theta}(\theta, x)dx \\ &\quad + \frac{d\hat{x}_B}{d\theta}\underbrace{\left[u_A(\theta, \hat{x}_B) - u_{AB}\right]}_{=0} - \frac{d\hat{x}_A}{d\theta}\underbrace{\left[u_B(\theta, \hat{x}_A) - u_{AB}\right]}_{=0} \\ &= 2\alpha\left(\hat{x}_B\right)^2 - 2\alpha\hat{x}_B(1-\theta)\frac{d\hat{x}_B}{d\theta},\end{aligned} \tag{11}$$

where we have used the fact that $\hat{x}_B = 1 - \hat{x}_A$ in the symmetric equilibrium.

The first term in Equation (11) represents a direct effect of increasing the level of interoperability for a given demand structure. A higher level of interoperability increases the quality of off-net interactions between the single-homers of each platform, in volume $2(\hat{x}_B)^2$. The second term in Equation (11) represents an indirect effect of a change in the level of interoperability. As the level of interoperability increases, the mass of single-homers on each platform increases. This represents a loss of utility from interactions for the single-homers on both platforms, since single-homers can only be reached by other single-homers through interoperability. The result in Lemma 4 then follows from the fact that the first effect always dominates the second:

$$\frac{dCS}{d\theta} = \frac{2\alpha(\hat{x}_B)^2 t}{t + \alpha(1-\theta)} > 0.$$

We find that producer surplus, defined as the sum of advertiser surplus and platforms' profits, also increases with the level of interoperability. This is because, first, advertisers have zero surplus in equilibrium. Their surplus from advertising to single-homers is fully extracted by the platforms, which have monopoly power. Then a multi-homer is worth $\sigma\tau^*$, but each platform charges advertisers $\sigma\tau^*/2$ for multi-homers, leaving advertisers with zero surplus.[20] Since the platforms' profits increase with the level of interoperability, the producer surplus increases with the level of interoperability.

Therefore, total welfare increases with the level of interoperability. So:

Proposition 2 (Regulator's interoperability choice). *A welfare-maximizing regulator sets the maximum level of interoperability $\theta^W = 1$. Therefore, the equilibrium level of interoperability always coincides with the social optimum.*

Therefore, in a symmetric environment, there should be no concern about the level of interoperability chosen by the platforms.

However, this conclusion becomes more nuanced when the costs of implementing interoperability are taken into account. Suppose that each platform incurs a fixed cost F to implement interoperability, independent of the level of interoperability.[21] In this case, perfect interoperability arises in equilibrium if and only if $F < \overline{F}^{\Pi} \equiv \Pi_{\theta=1} - \Pi_{\theta=0}$, whereas perfect interoperability is socially optimal whenever $F < \overline{F}^{W} \equiv \left(W_{\theta=1} - W_{\theta=0}\right)/2$, with $\overline{F}^{W} > \overline{F}^{\Pi}$.[22] Thus, the equilibrium level of interoperability may be too low compared to the social optimum when implementation costs are intermediate. This is because, when choosing its level of interoperability, a platform does not internalize the positive effects of interoperability on consumer surplus.

4 | Platform Asymmetry

In this section, we extend the baseline model by introducing an asymmetry between the platforms. We want to study the incentives of a large and a small platform to interoperate, and how the asymmetry between them affects these incentives.

Specifically, we assume that there is a mass $\delta > 0$ of users located at $y_A = 0$. In equilibrium, these users will always choose to single-home on A. So, we refer to them as the *installed base* of platform A. Because of this installed base, we refer to platform A as the large platform and platform B as the small platform.

Again, we proceed backwards, starting with the last stage.

4.1 | Consumer Homing Decisions

Once they have joined their platform(s), users spend time communicating on-net and off-net, as described in Section 3.1. We now turn to their homing decisions.

For the moment, we assume that the installed base users join platform A; we will verify that this is indeed true in equilibrium. We can now focus on the decision of the mass 1 of consumers located on the semi-open interval $(0, 1]$, which we call the *competitive segment*. A consumer located at $x \in (0, 1]$ has the following expected utility from single-homing on A and B:

$$u_A = v_0 + \alpha(r_A^e)\left[x_A^e + \delta + \theta(1 - x_A^e)\right] - tx,$$
$$u_B = v_0 + \alpha(r_B^e)\left[1 - x_B^e + \theta(x_B^e + \delta)\right] - t(1 - x).$$

By single-homing on platform A, a consumer can interact with the installed base of that platform with the full set of features. However, consumers single-homing on platform B can only interact with A's installed base via interoperability, so with a degraded quality of interaction.

We assume without loss of generality that $r_A^e \le r_B^e$ and we focus, as above, on the case where $\theta\alpha(r_A^e) \le \alpha(r_B^e)$, meaning that multi-homers use platform B to communicate with the single-homers of platform B. This condition will be satisfied in equilibrium. Using similar reasoning as in the baseline case, we obtain the utility derived by a multi-homing user:

$$u_{AB} = v_0(1+\eta) - t + \alpha(r_A^e)(x_A^e + \delta) + \alpha(r_B^e)(1 - x_A^e). \tag{12}$$

A consumer located at x joins platform A in addition to platform B if and only if $u_{AB} - u_B \ge 0$, that is, if $x \le \hat{x}_A$, where

$$\hat{x}_A = \frac{\eta v_0 + \left(\alpha(r_A^e) - \alpha(r_B^e)\right)(x_A^e + \delta) + (1-\theta)\alpha(r_B^e)(x_B^e + \delta)}{t}. \tag{13}$$

Similarly, a consumer located at x joins platform B in addition to platform A if and only if $u_{AB} - u_A \ge 0$, that is, if $x \ge \hat{x}_B$, where

$$\hat{x}_B = 1 - \frac{\eta v_0 + (1 - x_A^e)\left(\alpha(r_B^e) - \theta\alpha(r_A^e)\right)}{t}. \tag{14}$$

In the fulfilled expectations equilibrium, we must have $x_i^e = \hat{x}_i$. Replacing x_i^e for $\hat{x}_i$, we solve the system of equations given by (13) and (14) for $\hat{x}_A$ and $\hat{x}_B$. Denoting $\lambda(r_A^e, r_B^e) = \alpha(r_A^e) - \alpha(r_B^e)$ and $\psi(r_A^e, r_B^e) = \alpha(r_B^e) - \theta\alpha(r_A^e)$, we obtain:

$$\hat{x}_A(r_A^e, r_B^e) = \frac{\eta v_0[t - (1-\theta)\alpha(r_B^e)] + (1-\theta)\alpha(r_B^e)[t - \psi(r_A^e, r_B^e)] + \delta t[\lambda(r_A^e, r_B^e) + (1-\theta)\alpha(r_B^e)]}{t[t - \lambda(r_A^e, r_B^e)] - (1-\theta)\alpha(r_B^e)\psi(r_A^e, r_B^e)},$$

$$\hat{x}_B(r_A^e, r_B^e) = \frac{[t - \lambda(r_A^e, r_B^e) - \eta v_0][t - \psi(r_A^e, r_B^e)] + \eta v_0 \phi(r_A^e, r_B^e) + \delta[\lambda(r_A^e, r_B^e) + (1-\theta)\alpha(r_B^e)]\psi(r_A^e, r_B^e)}{t[t - \lambda(r_A^e, r_B^e)] - (1-\theta)\alpha(r_B^e)\psi(r_A^e, r_B^e)}.$$

Provided that $0 \le \hat{x}_B \le \hat{x}_A \le 1$, the numbers of single-homers and multi-homers on the competitive segment are then given by $SH_A(r^e_A, r^e_B) \equiv \hat{x}_B(r^e_A, r^e_B)$, $SH_B(r^e_A, r^e_B) \equiv 1 - \hat{x}_A(r^e_A, r^e_B)$ and $MH(r^e_A, r^e_B) \equiv \hat{x}_A(r^e_A, r^e_B) - \hat{x}_B(r^e_A, r^e_B)$.

4.2 | Advertising Decisions

In Stage 2, platforms set their price per ad, and advertisers decide on which platform(s) to buy ad space. The following lemma characterizes the equilibrium in this stage of the game.

Lemma 5 (Advertising equilibrium with platform asymmetry). *There exists a unique equilibrium where platforms set the price per ad* $p_A = p^{SH}_A(\delta + SH_A(1,1)) + p^{MH}_A MH(1,1)$ *and* $p_B = p^{SH}_B SH_B(1,1) + p^{MH}_B MH(1,1)$, *with* $p^{SH}_A = \sigma\tau^*(\delta + \hat{x}_A + \theta(1-\hat{x}_A))$, $p^{MH}_A = \sigma\tau^*(\delta + (\hat{x}_A + \hat{x}_B)/2)$, $p^{SH}_B = \sigma\tau^*(1 - \hat{x}_B + \theta(\hat{x}_B + \delta))$ *and* $p^{MH}_B = \sigma\tau^*(1-(\hat{x}_A+\hat{x}_B)/2)$. *Each advertiser places an ad on each platform, and users rationally expect* $r^e_i = 1$ *for* $i = A, B$. *In equilibrium, the platforms' demands in the competitive segment are as follows*:

$$SH_A(1,1) = \hat{x}_B(1,1) = \frac{t - \eta v_0}{t + (1-\theta)\alpha} + \delta \frac{(1-\theta)^2\alpha^2}{t^2 - (1-\theta)^2\alpha^2},$$

$$SH_B(1,1) = 1 - \hat{x}_A(1,1) = \frac{t - \eta v_0}{t + (1-\theta)\alpha} - \delta \frac{(1-\theta)\alpha t}{t^2 - (1-\theta)^2\alpha^2},$$

$$MH(1,1) = \hat{x}_A(1,1) - \hat{x}_B(1,1) = \frac{2\eta v_0 + \alpha(1-\theta)(1+\delta) - t}{t + (1-\theta)\alpha}.$$

For simplicity, as in the baseline case, we omit the arguments of $\hat{x}_i$, SH_i, and MH in the rest of the analysis.

Platform asymmetry and the level of interoperability affect the ad prices of the small and large platforms, as shown in the following lemma:

Lemma 6 (Platforms' ad prices with platform asymmetry). *The large platform can charge a higher price for single-homers and multi-homers than the small platform* ($p^s_A > p^s_B$ *for* $s = SH, MH$). *The price difference increases with the installed base, but decreases with the level of interoperability* ($\partial(p^s_A - p^s_B)/\partial\delta > 0$ *and* $\partial(p^s_A - p^s_B)/\partial\theta < 0$ *for* $s = SH, MH$).

Thus, a higher level of interoperability θ reduces the difference in ad prices between the large and small platforms and levels the playing field in the advertising market, though not completely. Interestingly, as interoperability becomes perfect ($\theta \to 1$), the difference in ad prices becomes 0 for single-homers, while it remains strictly positive for multi-homers. This is because the incremental value that platform A can charge for multi-homers includes the incremental value of its installed base.

As in the baseline case, we are interested in market structures in the competitive segment with both single-homers and multi-homers, that is, where in equilibrium, $SH_i > 0$ for $i = A, B$ and $MH > 0$, for all $\theta \in [0,1]$. This is true if:[23]

Assumption 3 (Partial multihoming with platform asymmetry).

$$\frac{t}{2} < \eta v_0 < t - \frac{\delta t \alpha}{t - \alpha}$$

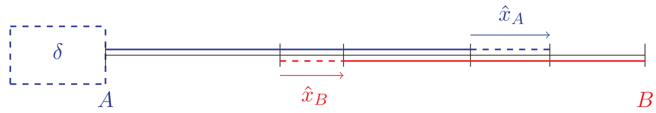

FIGURE 2 | Asymmetric shift in the composition of demand induced by an increase in δ. *Notes:* [Colour figure can be viewed at wileyonlinelibrary.com]

We assume that $\delta < \delta^{\max} = \frac{t-\alpha}{2\alpha}$ to ensure that there are values of ηv_0 that satisfy this assumption.[24] Under Assumption 3, we have $\hat{x}_B > 0$, so the installed base users all join platform A in equilibrium.

The following lemma characterizes how the installed base advantage of platform A affects the demand of each platform and the composition of demand in the competitive segment.

Lemma 7 (Effect of installed base on consumer demand). *A larger installed base δ leads to an increase in the demand for platform A and a decrease in the demand for platform B in the competitive segment. Single-homing on A increases, while single-homing on B decreases. Multi-homing increases.*

Intuitively, the installed base advantage of platform A allows the large platform to attract more demand than the small platform in the competitive segment due to network effects.

The installed base also affects the composition of demand. Consider the variations in the location of the marginal multi-homers given by differentiating Equations (13) and (14) at $r^e_i = 1$ with respect to δ:

$$\frac{d\hat{x}_A}{d\delta} = \frac{\alpha(1-\theta)}{t}\left[1 + \frac{d\hat{x}_B}{d\delta}\right]$$
$$\frac{d\hat{x}_B}{d\delta} = \frac{\alpha(1-\theta)}{t}\frac{d\hat{x}_A}{d\delta}.$$

First, an increase in the size of the installed base δ makes second-homing on platform A more attractive to interact with the installed base of that platform. Thus, the marginal consumer indifferent between multi-homing and single-homing on B, located at $\hat{x}_A$, shifts to the right, reducing the exclusive user base of platform B. This, in turn, reduces the incentive to second-home on platform B. Therefore, the marginal consumer indifferent between multi-homing and single-homing on A, located at $\hat{x}_B$, also shifts to the right, though less so than for $\hat{x}_B$ (see Figure 2 for an illustration). Since this increases the size of platform A's exclusive user base, it further increases the incentives to second-home on that platform, and so on. Assumption 1 ensures that the market does not tip and that both platforms remain active in equilibrium. Overall, a larger installed base increases the single-homing demand of the large platform and decreases that of the small platform.

4.3 | Platforms' Interoperability Decisions

In Stage 1, platforms decide non-cooperatively on their level of interoperability. We begin by characterizing the impact of interoperability on the consumer demand for each platform in the competitive segment.

Lemma 8 (Interoperability and consumer demand with platform asymmetry). *An increase in the level of interoperability*:

- *reduces consumer multi-homing*;
- *decreases the single-homing demand of platform A if $\theta < \hat{\theta}$ and increases it if $\theta > \hat{\theta}$, where $\hat{\theta} \in [0, 1)$ and $\hat{\theta} > 0$ if δ is high enough*;
- *increases the single-homing demand of platform B.*

Similar to the baseline case, interoperability reduces multi-homing when platforms are asymmetric. However, an increase in interoperability affects the demand composition of the small and large platforms in the competitive segment differently. While the single-homing demand of the small platform always increases with the level of interoperability, the single-homing demand of the large platform may decrease at low levels of interoperability and increase at higher levels. To understand the intuition, consider the variations of the location of the marginal users $\hat{x}_A$ and $\hat{x}_B$, given by Equations (13) and (14), with respect to the level of interoperability θ, at $r_i^e = 1$ and $x_i^e = \hat{x}_i$:

$$\frac{d\hat{x}_A}{d\theta} = \frac{\alpha}{t}\left[-(SH_A + \delta) + (1-\theta)\frac{d\hat{x}_B}{d\theta}\right], \tag{15}$$

$$\frac{d\hat{x}_B}{d\theta} = \frac{\alpha}{t}\left[SH_B + (1-\theta)\frac{d\hat{x}_A}{d\theta}\right]. \tag{16}$$

As in the symmetric case, the first terms in Equations (15) and (16) represent a direct effect of a higher level of interoperability, taking the demand composition as given. Since it increases the quality of off-net interactions, a higher level of interoperability reduces the incentive to multi-home and pushes to single-home. Also, since we have $SH_A + \delta > SH_B$, the increase in single-homing is greater for the small platform (B) than for the large one (A).

The second terms in Equations (15) and (16) represent an (opposite) indirect effect: since a higher level of interoperability increases the single-homing demand of each platform, it also makes multi-homing more attractive. For example, an increase in the single-homing demand of platform B ($d\hat{x}_A/d\theta \leq 0$) increases the incentives of the single-homing users of platform A to multi-home.

Since the direct effect is larger for the small platform than for the large platform, this negative feedback loop is more significant for the large platform. Therefore, a higher level of interoperability leads to a greater increase in single-homing demand for the small platform than for the large one. For low levels of interoperability, the indirect effect plays a more important role, and the large platform's single-homing demand may actually decrease with a higher level of interoperability.

Since the total demand of a platform decreases with the single-homing demand of the competing platform, Lemma 8 implies that:

Corollary 2. *An increase in the level of interoperability decreases platform A's total demand in the competitive segment. Platform B's total demand increases if $\theta < \hat{\theta}$ and decreases otherwise.*

Therefore, interoperability reduces the total demand of the large platform in the competitive segment, but it does not always benefit the small platform. While the single-homing demand of the small platform increases with interoperability, the number of multi-homers decreases, and thus its total demand may either increase or decrease. However, we find that the difference in total demand between the large and the small platforms decreases with the level of interoperability. Hence, interoperability levels the playing field between them in terms of network effects.

We now turn to the non-cooperative interoperability decisions of the platforms. In Stage 1, the platforms' profits are:

$$\Pi_A(\theta_A, \theta_B) = p_A^{SH}(\theta)(\delta + \hat{x}_B) + p_A^{MH}(\theta)\big(\hat{x}_A - \hat{x}_B\big)$$
$$\Pi_B(\theta_A, \theta_B) = p_B^{SH}(\theta)\big(1 - \hat{x}_A\big) + p_B^{MH}(\theta)\big(\hat{x}_A - \hat{x}_B\big),$$

with $\theta = \min\{\theta_A, \theta_B\}$.

We begin by characterizing the level of interoperability that each platform would prefer (i.e., if it could choose the level of interoperability θ between them):

Proposition 3 (Platforms' interoperability choices with platform asymmetry). *The small platform always prefers perfect interoperability ($\theta_B^* = 1$). Besides, there is a threshold δ_{Π_A} such that the large platform prefers perfect interoperability ($\theta_A^* = 1$) if $\delta < \delta_{\Pi_A}$, and no interoperability ($\theta_A^* = 0$) if $\delta > \delta_{\Pi_A}$.*

Thus, if the platform asymmetry is large enough, the large platform will be less willing to interoperate and the small platform more willing to interoperate compared to the baseline case with symmetric platforms.

To understand this result, we examine how platform asymmetry affects the three key effects of interoperability on platform profits that we introduced in the baseline case: the market power effect, the total viewership effect, and the usage intensification effect.[25]

In the baseline case, the market power effect arises because interoperability increases the single-homing demand for each platform. This benefits the platforms because they can charge higher ad prices for (exclusive) single-homers compared to multi-homers. Under platform asymmetry, platforms still charge higher prices for single-homers, and this effect is even more pronounced for the smaller platform. Moreover, the increase in single-homing demand is larger for the smaller platform than for the larger one (Lemma 8). Consequently, the market power effect is stronger for the smaller platform, and may even be negative for the larger platform.

The total viewership effect in the baseline case reflects how interoperability reduces the total demand of each platform by decreasing multi-homing. With platform asymmetry, the reduction in total demand due to interoperability is more significant for the larger platform than for the smaller platform. In addition, because the larger platform can charge more for multi-homers, it is more negatively affected by the total viewership effect than the smaller platform.

Finally, the usage intensification effect in the baseline case captures how interoperability increases platform usage, thereby increasing exposure to advertising and benefiting the platforms. Under platform asymmetry, this effect is also stronger for the smaller platform than for the larger one. This is because interoperability helps level the playing field in the advertising market (Lemma 6).

Since the small platform has a higher incentive to interoperate than the large platform, the equilibrium level of interoperability, $\theta^\star = \min\{\theta_A^\star, \theta_B^\star\}$, is always determined by the large platform (i.e., equal to $\theta_A^\star$). Therefore, we have:

Corollary 3. *In equilibrium, the level of interoperability is $\theta^\star = 1$ if $\delta \le \delta_{\Pi_A}$ and $\theta^\star = 0$ otherwise.*

Therefore, if the large platform's installed base advantage is not too strong ($\delta \le \delta_{\Pi_A}$), the equilibrium outcome is the same as in the symmetric case. Otherwise, if the large platform has a large installed base ($\delta \ge \delta_{\Pi_A}$), interoperability will not arise in equilibrium.

4.4 | Welfare Analysis

We now discuss how interoperability affects the surplus of market participants and the optimal choice of interoperability by a welfare-maximizing regulator.

Taking into account the surplus of the installed base users, consumer surplus is given by:

$$CS(\theta) = \delta u_A(\theta, 0) + \int_0^{\hat{x}_B(\theta)} u_A(\theta, x)dx + \int_{\hat{x}_B(\theta)}^{\hat{x}_A(\theta)} u_{AB}dx + \int_{\hat{x}_A(\theta)}^{1} u_B(\theta, x)dx. \quad (17)$$

We find that:

$$\frac{dCS}{d\theta} = 2\alpha\left(\delta + \hat{x}_B\right)(1 - \hat{x}_A) + \alpha(1 - \theta)\left[(\delta + \hat{x}_B)\frac{d\hat{x}_A}{d\theta} - (1 - \hat{x}_A)\frac{d\hat{x}_B}{d\theta}\right]. \quad (18)$$

As in the symmetric case, the first term in Equation (18) represents the gain in consumer surplus from a higher quality of interaction between single-homers, where the volume of these interactions is equal to $2(\delta + \hat{x}_B)(1 - \hat{x}_A)$.

The second term in Equation (18) represents a change in utility for the single-homers of both platforms as the marginal consumers located at $\hat{x}_A$ and $\hat{x}_B$ shift toward single-homing (loss) or multi-homing (gain). A priori, the sign of this term is ambiguous, since $d\hat{x}_B/d\theta$ can be either positive or negative. However, we find that it is always negative. Indeed, according to Lemma 8, consumer multi-homing decreases as interoperability increases. Thus, we have $d\hat{x}_A/d\theta < d\hat{x}_B/d\theta$, with $d\hat{x}_A/d\theta < 0$. As the mass of platform A's single-homers ($\delta + \hat{x}_B$) is higher than platform B's ($1 - \hat{x}_A$),[26] this second term represents a loss of utility.

Overall, how consumers value interoperability depends on the degree of platform asymmetry:

Lemma 9 (Consumer surplus with platform asymmetry). *Under platform asymmetry, there is a threshold δ_{CS} such that consumer surplus is maximized with perfect interoperability ($\theta = 1$) if $\delta < \delta_{CS}$, and with no interoperability ($\theta = 0$) otherwise. This threshold is lower than the maximum allowed value of δ if $(t - \eta v_0)/t$ is low enough.*

Thus, like the platforms themselves, consumers may prefer no interoperability when the platforms are highly asymmetric. However, we find that:

Proposition 4 (Misalignment of interoperability choices with platform asymmetry). *Under platform asymmetry, the level of interoperability resulting from the market equilibrium is (weakly) too low from the users' point of view.*

Indeed, the threshold below which interoperability arises in equilibrium is lower than the corresponding threshold for consumer surplus ($\delta_{\Pi_A} < \delta_{CS}$). Moreover, both thresholds decrease when network effects become stronger relative to the degree of differentiation (i.e., when α is larger relative to t) and/or when the additional standalone benefits from multi-homing, ηv_0, are greater.

Since the equilibrium level of interoperability is (weakly) too low for consumers and the small platform, it is also (weakly) too low from a total welfare point of view.[27] This is shown formally by the following result:

Proposition 5 (Total welfare with platform asymmetry). *Under platform asymmetry, there exists a threshold δ_W such that a welfare-maximizing regulator sets the maximum level of interoperability $\theta^W = 1$ if $\delta < \delta_W$, and the maximum level of interoperability $\theta^W = 0$ otherwise. This threshold is higher than the threshold for perfect interoperability in the market equilibrium ($\delta_{\Pi_A} < \delta_W$), so the equilibrium level of interoperability is (weakly) too low from a welfare point of view.*

Therefore, in asymmetric markets, regulatory intervention can be necessary to achieve perfect interoperability, as it may not emerge as the market equilibrium, even when it is socially desirable. Conversely, if platforms implement interoperability voluntarily, which happens if the degree of platform asymmetry is not too pronounced, it is also efficient from a social welfare perspective.

However, the absence of interoperability in the market equilibrium does not necessarily call for regulatory intervention. Although interoperability levels the playing field between platforms, it may negatively affect consumer surplus and total surplus in highly asymmetric markets. Specifically, mandating interoperability is socially desirable when $\delta \in (\delta_{\Pi_A}, \delta_W)$, but it harms welfare when $\delta > \delta_W$, that is, when the market is highly asymmetric.

Introducing a fixed cost to implement interoperability creates additional distortions. As in the baseline case, when the implementation cost is intermediate, interoperability may not emerge when $\delta < \delta_{\Pi_A}$ even though it could be socially optimal. This is because platform A does not take into account the effect of interoperability on platform B, which may benefit from implementing perfect interoperability, and on consumer surplus. Furthermore, the smaller platform may face a higher fixed cost of implementing interoperability than the larger platform. For instance, suppose interoperability is costless for the larger platform ($F_A = 0$) but costly for the smaller one ($F_B > 0$). If $\delta < \delta_{\Pi_A}$ and $\Pi_B|_{\theta=1} - \Pi_B|_{\theta=0} < F_B$, interoperability arises in equilibrium because the larger platform has an incentive to implement it, but it harms the smaller platform because of high implementation costs.

5 | Conclusion

In this paper, we have studied the incentives of ad-financed platforms to make their services interoperable and how these incentives compare to the social optimum. We showed that interoperability affects the demand composition of the platforms by reducing multi-homing and increasing single-homing. As multi-homers are less valuable to advertisers than single-homers, interoperability increases the market power of platforms over advertisers. Additionally, it stimulates the engagement of single-homers due to higher-quality interactions, increasing their exposure to advertising. However, interoperability also reduces the total viewership that the platforms can monetize on the advertiser side. Overall, the first two positive effects outweigh the last negative effect, and perfect interoperability emerges in an unregulated environment with symmetric platforms. This equilibrium outcome coincides with the social optimum when the costs of implementing interoperability are not too high.

In markets where a large, dominant platform competes with a smaller platform, interoperability levels the playing field by reducing the difference in network effects between them. It also levels the playing field in the advertising market by reducing the difference in ad prices between the small and large platforms. As a result, the large platform is less willing to make its service interoperable than the small platform. We then find that interoperability does not emerge in equilibrium if the installed base of the large platform is sufficiently large. However, mandating interoperability between the asymmetric platforms is not always socially optimal.

Our results are relevant for assessing the potential impact of interoperability obligations for ad-financed platforms, such as the horizontal interoperability obligation for messaging services implemented in the Digital Markets Act in the European Union, or a possible extension of these obligations to social networks (e.g., TikTok or Instagram). We show that it is important to consider the multi-sided nature of these platforms, as interoperability on the consumer side has implications on the advertiser side through the changes in demand composition and consumer engagement that it induces. Thus, a regulator should balance the consumer benefits of interoperability against the potentially adverse effects on other market participants. In particular, we show that interoperability may increase the market power of platforms in the advertising market, which may raise antitrust concerns.

In our framework, a merger between the platforms would result in monopoly pricing in the advertising market for all consumers. The merged entity would thus have no incentive to implement interoperability between its two services. In practice, messaging platforms owned by the same company (such as Facebook Messenger and WhatsApp) are not interoperable. Hence, when assessing mergers in digital markets, antitrust authorities should take into account the reduced incentives for interoperability as a potential adverse effect.

In this paper, we focused on interoperability between ad-funded platforms. However, platforms may adopt a variety of business models, and some may instead be user-funded.[28] In the case of messaging services, for instance, some platforms are user-funded—such as Threema,[29] or Apple's iMessage,[30] which are financed directly by their users rather than through advertising.[31] If an ad-funded platform competes with a user-funded one, the former would enjoy a monopoly over multi-homing users in the advertising market. In this case, the market power effect would vanish, whereas the total viewership and usage intensification effects would remain at play.[32]

Finally, in the paper, we have abstracted away from the implementation issues raised by interoperability.[33] A key trade-off is between standardization, which requires firms to adopt a common interoperability standard, and reliance on proprietary interfaces (APIs), where firms implement each other's interfaces. Standardization facilitates effective interoperability, but it requires industry-wide agreement, which takes time and involves uncertainty. Proprietary access, on the other hand, leaves control with the gatekeeper, which may have the ability and incentive to restrict or degrade access.[34] A second trade-off concerns security and privacy. Interoperability can expand attack vectors, reduce security, and alter firms' incentives to invest in cybersecurity.[35] Interoperability may also create tension between privacy and security, as secure communication often requires exchanging data or metadata to verify authenticity.[36] These questions are fruitful directions for future research on interoperability in digital markets.

Acknowledgements

We thank the Editor, Matthew Mitchell, and two anonymous referees for their valuable comments. We also thank Marie-Laure Allain, Enrick Arnaud-Joufray, Germain Gaudin, Axel Gautier, Yassine Lefouili, José-Luis Moraga, Michelangelo Rossi, Wilfried Sand-Zantman, and Basile Vidalenc for their helpful comments. Finally, we thank the audiences at the 13th Paris Conference on Digital Economics, the 5th CESifo Doctoral Workshop on the Economics of Digitization, the XII NERI Workshop, the Bergen Competition Policy Conference 2022, the 70th Congress of the French Economic Association, the 11th IBEO Workshop, the 8th AFREN Digital Economics Summer School, and the EARIE annual meetings 2022 and 2025. The views expressed are those of the authors and do not reflect the official position of the European Commission where Guillaume Thébaudin was previously employed.

Endnotes

[1] Fortune, *Larry Page on Google*, December 11, 2012.

[2]Regulation (EU) 2022/1925 of the European Parliament and of the Council of 14 September 2022 on contestable and fair markets in the digital sector, http://data.europa.eu/eli/reg/2022/1925/oj.

[3]See https://www.congress.gov/bill/117th-congress/house-bill/3849/text.

[4]In their model, there are two types of consumers with either a low or a high valuation of network benefits. When considering multi-homing, they focus on equilibria where low types single-home, while high types multi-home.

[5]See also Gabszewicz, Laussel and Sonnac [23].

[6]Examples of ad-supported messaging services include Facebook Messenger, Telegram, Snapchat, Kik, Line, Viber, and WeChat.

[7]Alternatively, our modeling approach could be reinterpreted to capture platforms that collect and monetize user data. In this case, consumers suffer from nuisance from data monetization.

[8]Introducing an (engineering) cost to implement interoperability would not qualitatively change our results, but would lead to a lower level of interoperability in equilibrium.

[9]Messaging services often offer stand-alone features in addition to communication capabilities, such as photo editing (Instagram), AI chatbots (Snapchat), or access to broadcast channels (Telegram). The stand-alone benefit may also reflect the fear of being left out if one does not join (Bursztyn, Handel, Jiménez-Duran and Roth [24]).

[10]In principle, single-homers on platform i could interact with multi-homers on platform j via interoperability. However, in our model, on-net interactions always deliver higher interaction benefits than off-net interactions via interoperability, as $\phi \leq 1$.

[11]This utility function has been used in the telecommunications context by Laffont, Rey and Tirole [25, 26] and Nevo, Turner and Williams [27], among others. As we will see below, the parameter β represents the elasticity of time spent on communication to the level of advertising.

[12]We could consider additional learning costs for a multi-homing consumer, or costs of maintaining and managing contacts across platforms. However, this would not affect the analysis.

[13]If firm j chooses $\phi_j < \min\{\phi_A^\star, \phi_B^\star\}$, firm i has no incentive to choose a ϕ_i greater than ϕ_j, as the resulting level of interoperability will remain at ϕ_j. For this reason, there can be multiple equilibria with $\phi < \min\{\phi_A^\star, \phi_B^\star\}$. Note that the assumption we make is equivalent to selecting the Pareto-dominant equilibrium when there are multiple equilibria.

[14]The second-order condition is satisfied as $u''(\tau) \leq 0$.

[15]Our results are robust if platforms and advertisers can identify whether consumers are single-homers or multi-homers. In this case, platforms could charge different prices per impression for single-homers and multi-homers. Each platform i would charge for single-homers a price of $p_i^{SH} = \sigma\tau^{SH}$ and for multi-homers a price of $p_i^{MH} = \sigma\tau^*/2$, leading to full participation from advertisers. Thus, platform i would derive the same ad revenue $p_i = p_i^{SH} SH_i(1,1) + p_i^{MH} MH(1,1) = \sigma\tau^{SH} SH_i(1,1) + \sigma\tau^* MH(1,1)/2$, as with a fixed ad price.

[16]If advertisers are heterogeneous (e.g., because they promote products of different qualities), those with a high valuation for an impression will tend to multi-home, while those with a lower valuation will tend to single-home. This might give rise to asymmetric equilibria, where one platform attracts a larger consumer demand than its rival (see Anderson and Jullien [28]).

[17]In Appendix B, we show that under Assumptions 1 and 2, there is a unique equilibrium with market-sharing and partial multi-homing.

[18]Indeed, we have $\frac{\partial \tau^{SH}}{\partial \theta} = \frac{t(t-\eta v_0)\tau^*}{[t+\alpha(1-\theta)]^3} > 0$.

[19]In [14], increasing interoperability increases total demand due to higher quality interactions. In our case, it reduces the total demand of each platform due to decreased multi-homing, while we do not consider the possibility of market expansion since the market is always fully covered.

[20]This is due to our assumption that the probability ϵ that a consumer will pay attention to an ad increases linearly with the amount of time τ of exposure to the ad across all platforms. If instead we assume that ϵ is strictly concave in τ, advertisers will derive a positive surplus.

[21]We could also consider that the implementation cost increases with the level of interoperability. However, this would not alter our point.

[22]Perfect interoperability is socially optimal if $W_{\theta=1} - W_{\theta=0} > 2F$. Thus, we have $\overline{F}^W = (W_{\theta=1} - W_{\theta=0})/2 = \Pi_{\theta=1} - \Pi_{\theta=0} + (CS_{\theta=1} - CS_{\theta=0})/2 > \Pi_{\theta=1} - \Pi_{\theta=0} = \overline{F}^\Pi$.

[23]In Appendix C, we show that under Assumptions 1 and 3, there is a unique equilibrium with market-sharing and partial multi-homing. As in the baseline case, the equilibrium is stable if $t > (1-\theta)\alpha$, which is true for all $\theta \in [0,1]$ if $t > \alpha$ (Assumption 1).

[24]For larger values of δ, there can be equilibria where platform B has no single-homers, only multi-homers (i.e., $\hat{x}_A = 1$ and $\hat{x}_B > 0$). In this case, we find that the number of multi-homers is independent of the level of interoperability. Therefore, interoperability does not affect firms' profits.

[25]See Appendix D for details.

[26]Indeed, we have $\delta + \hat{x}_B - (1 - \hat{x}_A) = \delta t/(t - \alpha(1-\theta)) > 0$.

[27]As in the baseline case, advertisers derive zero surplus in equilibrium. See the discussion in Footnote 20.

[28]On competition between ad-funded and user-funded platforms, see, for example, Etro [29] and Allain, Bourreau and Darlas [30].

[29]Individual users pay a one-time fee of six euros to use the app, while corporate users are charged three euros or more per month per user.

[30]The messaging service is bundled with the purchase of Apple devices.

[31]WhatsApp used to charge a $0.99 annual subscription fee after the first year of free use. However, in January 2016, the fee was discontinued, and the app became free. In mid-June 2025, WhatsApp announced that it would start showing ads in its app.

[32]See also Dhakar and Yan [31], who study the impact of interoperability in a model of competition between a low-privacy ad-funded platform and a high-privacy user-funded platform.

[33]See Bourreau and Kraemer [32] for a discussion of some of the implementation issues for messaging services.

[34]On cost-increasing and demand-reducing sabotage tactics in vertically integrated industries, see, for example, Mandy and Sappington [33].

[35]For example, Comino, Fedele and Manenti [34] show that interoperability can promote security investments at low levels but may discourage them at high levels.

[36]See, for example, the concerns raised about the Meta Interoperability proposal based on API access, "Detailed technical briefing: The Case for XMPP—Why Meta Must Embrace True Messaging Interoperability", Key Concern 3, https://xmpp.org/announcements/open-letter-meta-dma/technical-briefing/.

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Appendix A

Proofs

Proof of Lemma 1. When a multi-homing consumer interacts with a single-homer, she can either use the same platform as the single-homer or the other platform via interoperability. To interact with a single-homer on platform A, the multi-homing consumer will always use platform A, as we have $\alpha(r_A^e) \geq \theta\alpha(r_B^e)$ because $\alpha'(r) < 0$, $r_A^e \leq r_B^e$, and $\theta \leq 1$. When communicating with a single-homer on B, the multi-homing consumer will use platform B if $\alpha(r_B^e) \geq \theta\alpha(r_A^e)$, and platform A otherwise. Finally, when communicating with another multi-homer, the multi-homing consumer will always use platform A, as $r_A^e \leq r_B^e$ implies that $\alpha(r_A^e) \geq \alpha(r_B^e)$. Therefore, the multi-homer will always use platform A except when $\alpha(r_B^e) \geq \theta\alpha(r_A^e)$, in which case the multi-homer will use platform B to interact with platform B's single-homers. □

Proof of Lemma 2. We proceed in two steps: We determine (1) the equilibrium ad prices for any given consumer expectations about ad levels, (2) the equilibrium where consumers make rational expectations about ad levels.

1. **Equilibrium ad prices for any given consumer expectations about ad levels**
   To determine the equilibrium ad prices for any given consumer expectations about ad levels, we begin by determining the value of single-homers and multi-homers to advertisers.

   Value of single-homers
   Consider a consumer who single-homes on platform A. She will spend an amount of time $\tau^*(r_A^e)$ for on-net communications, and $\hat{\tau}^*(r_A^e)$ for off-net communications. Thus,

the total amount of time she will spend on platform A is $\left[\hat{x}_A(r_A^e, r_B^e) + \theta(1 - \hat{x}_A(r_A^e, r_B^e))\right]\tau^*(r_A^e) \equiv \tau_A^{SH}(r_A^e, r_B^e)$. If platform A has only $SH_A(r_A^e, r_B^e)$ single-homing users, the total value generated by this platform for advertisers is $\sigma\tau_A^{SH}(r_A^e, r_B^e)SH_A(r_A^e, r_B^e)$. Similarly, if platform B has only $SH_B(r_A^e, r_B^e)$ single-homing users, the total value it generates for advertisers is $\sigma\tau_B^{SH}(r_A^e, r_B^e)SH_B(r_A^e, r_B^e)$, where $\tau_B^{SH}(r_A^e, r_B^e) \equiv \left[1 - \hat{x}_B(r_A^e, r_B^e) + \theta\hat{x}_B(r_A^e, r_B^e)\right]\tau^*(r_B^e)$.

Value of multi-homers

Now, consider a multi-homing consumer. We have to distinguish two cases, depending on whether $r_A^e = r_B^e$ or $r_A^e < r_B^e$ (remember that we have assumed that $r_A^e \leq r_B^e$).

Case 1: $r_A^e = r_B^e$. In this case, a multi-homer will be indifferent between using platform A or platform B to communicate with other multi-homers, and we assume that she will use A or B equally. She will thus spend a total amount of time on both platforms equal to $\tau^*(r_A^e)\left[\hat{x}_B + (\hat{x}_A - \hat{x}_B)/2\right] + \tau^*(r_B^e)\left[(\hat{x}_A - \hat{x}_B)/2 + (1 - \hat{x}_A)\right] = \tau^*(r_A^e)$. The total value of multi-homers for an advertiser who buys ad space on both platforms is then $\sigma\tau^*(r_A^e)MH(r_A^e, r_B^e)$. However, a platform can only charge the advertisers for the *incremental value* that it provides to them. For example, platform A can only charge the difference between the total value of multi-homers, $\sigma\tau^*(r_A^e)MH(r_A^e, r_B^e)$, and the value of multi-homers if the advertisers only buy ad-space on platform B, which is equal to $\sigma\tau^*(r_B^e)\left[(\hat{x}_A - \hat{x}_B)/2 + (1 - \hat{x}_A)\right]MH(r_A^e, r_B^e)$. Thus, the incremental value offered by platform A to advertisers is $\sigma\tau^*(r_A^e)\left[\hat{x}_B + (\hat{x}_A - \hat{x}_B)/2\right]MH(r_A^e, r_B^e) = \sigma\tau^*(r_A^e)MH(r_A^e, r_B^e)/2$, given that $\hat{x}_B = 1 - \hat{x}_A$ if $r_A^e = r_B^e$. Since this case is symmetric, this is also the incremental value offered by platform B to advertisers.

Case 2a: $r_A^e < r_B^e$ and $\theta\alpha(r_A^e) \leq \alpha(r_B^e)$. In this case, from Lemma 1, multi-homers use platform A to communicate with other multi-homers and single-homers of this platform, and platform B to communicate with platform B's single-homers. They thus spend a total amount of time on both platforms equal to $\tau^*(r_A^e)\hat{x}_A(r_A^e, r_B^e) + \tau^*(r_B^e)(1 - \hat{x}_A(r_A^e, r_B^e))$. Using a similar logic as above, the incremental value offered by platform A to advertisers is $\sigma\tau^*(r_A^e)\hat{x}_A(r_A^e, r_B^e)MH(r_A^e, r_B^e)$ and the incremental value offered by platform B is $\sigma\tau^*(r_B^e)(1 - \hat{x}_A(r_A^e, r_B^e))MH(r_A^e, r_B^e)$.

Case 2b: $r_A^e < r_B^e$ and $\theta\alpha(r_A^e) > \alpha(r_B^e)$. In this case, from Lemma 1, multi-homers always use platform A to communicate with other users. As a result, they spend a total amount of time on platform A equal to $\tau^*(r_A^e)$. Therefore, the incremental value offered by platform A to advertisers is $\sigma\tau^*(r_A^e)MH(r_A^e, r_B^e)$, while platform B offers no incremental value.

Equilibrium ad prices for any given consumer expectations about ad levels

We now assume that platforms have both single-homers and multi-homers, and we determine the equilibrium ad prices given consumer expectations about ad levels r_A^e and r_B^e, using the analysis above.

Case 1: $r_A^e = r_B^e$. In this case, there is a unique equilibrium where each platform prices at the incremental value, with $p_A = \sigma\tau_A^{SH}(r_A^e, r_B^e)SH_A(r_A^e, r_B^e) + \sigma\tau^*(r_A^e)MH(r_A^e, r_B^e)/2$ and $p_B = \sigma\tau_B^{SH}(r_A^e, r_B^e)SH_B(r_A^e, r_B^e) + \sigma\tau^*(r_B^e)MH(r_A^e, r_B^e)/2$. It is an equilibrium, because if firm j prices as such, it is a best response for firm i to do so as well. Any higher price would lead to zero demand from advertisers. Conversely, any lower price would not attract more advertisers. The equilibrium is also unique: since all platforms must sell ad space in any equilibrium, all advertisers must be on both platforms. If they are all on each platform, then platform i must be pricing at the *incremental value*.

Case 2a: $r_A^e < r_B^e$ and $\theta\alpha(r_A^e) \leq \alpha(r_B^e)$. Using a similar reasoning as in Case 1, in the unique equilibrium, platforms set the ad prices $p_A = \sigma\tau_A^{SH}(r_A^e, r_B^e)SH_A(r_A^e, r_B^e) + \sigma\tau^*(r_A^e)\hat{x}_A(r_A^e, r_B^e)MH(r_A^e, r_B^e)$ and $p_B = \sigma\tau_B^{SH}(r_A^e, r_B^e)SH_B(r_A^e, r_B^e) + \sigma\tau^*(r_B^e)(1 - \hat{x}_A(r_A^e, r_B^e))MH(r_A^e, r_B^e)$, and all advertisers buy ad space on both platforms.

Case 2b: $r_A^e < r_B^e$ and $\theta\alpha(r_A^e) > \alpha(r_B^e)$. Again, using a similar reasoning as in Case 1, in the unique equilibrium, platforms set the ad prices $p_A = \sigma\tau_A^{SH}(r_A^e, r_B^e)SH_A(r_A^e, r_B^e) + \sigma\tau^*(r_A^e)MH(r_A^e, r_B^e)$ and $p_B = \sigma\tau_B^{SH}(r_A^e, r_B^e)SH_B(r_A^e, r_B^e)$, and all advertisers buy ad-space on both platforms.

2. **Equilibrium ad prices with rational expectations about ad levels**
   In equilibrium, consumers must rationally anticipate the levels of advertising on the two platforms. In all the equilibria described above, all advertisers participate on both platforms. Given that there is a unit mass of homogeneous advertisers, the realized level of advertising in equilibrium—which we defined as the number of advertisers—is $r_A = r_B = 1$. Consequently, consumers must rationally anticipate $r_A^e = r_B^e = 1$, and Cases 2a and 2b, in which $r_A^e < r_B^e$, cannot arise in equilibrium. Thus, the unique equilibrium corresponds to Case 1, with $r_A^e = r_B^e = 1$. The equilibrium ad prices are then given by:

$$p_i = \sigma\tau_i^{SH}(1,1)SH_i(1,1) + \sigma\tau^*(1)MH(1,1)/2.$$

□

Proof of Lemma 3. Under Assumption 2, we have $dMH/d\theta < 0$, which gives the result. □

Proof of Proposition 1. We show that platforms' profits increase with the level of interoperability. Platform A's total advertising revenues are:

$$\Pi_A = \sigma\tau^*\left(\hat{x}_B\left(\hat{x}_A + \theta(1 - \hat{x}_A)\right) + (\hat{x}_A - \hat{x}_B) \times \frac{1}{2}\right).$$

Differentiating this profit function with respect to the level of interoperability θ, we find that

$$\frac{d\Pi_A}{d\theta} = \sigma\tau^*\frac{(t - \eta v_0)^2(t - (1 - \theta)\alpha)}{(t + (1 - \theta)\alpha)^3} > 0,$$

since $t > (1 - \theta)\alpha$ under Assumption 1. Thus, platform A's profit increases with the level of interoperability θ. Due to the symmetry of the model, the same holds for platform B. As $\theta_A^* = \theta_B^* = 1$, the equilibrium level of interoperability is $\theta^* = 1$. □

Proof of Lemma 5. Using similar reasoning as in Lemma 2, we find that platform A can charge advertisers the price $p_A^{SH} = \sigma\tau_A^{SH}(r_A^e, r_B^e)$ for reaching its $SH_A(r_A^e, r_B^e) + \delta$ single-homers, where $SH_i(r_A^e, r_B^e)$ represents platform i's number of single-homers in the competitive segment, with $\tau_A^{SH}(r_A^e, r_B^e) \equiv \left[\delta + \hat{x}_A(r_A^e, r_B^e) + \theta(1 - \hat{x}_A(r_A^e, r_B^e))\right]\tau^*(r_A^e)$. Platform B can charge $p_B^{SH} = \sigma\tau_B^{SH}(r_A^e, r_B^e)$ for reaching its $SH_B(r_A^e, r_B^e)$ single-homers, with $\tau_B^{SH}(r_A^e, r_B^e) \equiv \left[1 - \hat{x}_B(r_A^e, r_B^e) + \theta(\delta + \hat{x}_B(r_A^e, r_B^e))\right]\tau^*(r_B^e)$.

Now, consider a multi-homing consumer and suppose that $r_A^e = r_B^e$. Multi-homers always communicate with all $(1 + \delta)$ users with the highest quality of communication, hence spend a total of time $(1 + \delta)\tau^*(r_A^e)$. Therefore, the value of a multi-homer for an advertiser who buys ad space on both platforms is $\sigma(1 + \delta)\tau^*(r_A^e)$. If, instead, the advertiser single-homes on platform B, a multi-homer is exposed to its ads for the time spent communicating with platform B's single-homers, $SH_B(r_A^e, r_B^e)\tau^*(r_B^e)$, and half of the time spent communicating with multi-homers, $(MH(r_A^e, r_B^e)/2)\tau^*(r_B^e)$. Hence, the incremental value of advertising on platform A in addition to B is $\sigma(1 + \delta)\tau^*(r_A^e) - \sigma(SH_B(r_A^e, r_B^e) + MH(r_A^e, r_B^e)/2)\tau^*(r_B^e) = \sigma\tau^*(r_A^e)(\delta + SH_A(r_A^e, r_B^e) + MH(r_A^e, r_B^e)/2) \equiv \sigma\tau_A^{MH}(r_A^e, r_B^e)$, using the fact that $SH_A + SH_B + MH = 1$. Therefore, platform A sets its price for accessing multi-homing consumers, p_A^{MH}, equal to this incremental value. Hence, the price for ad space charged by the platform A is:

$$p_A = \underbrace{\sigma\tau_A^{SH}(r_A^e, r_B^e)}_{=p_A^{SH}}(\delta + SH_A(r_A^e, r_B^e)) + \underbrace{\sigma\tau_A^{MH}(r_A^e, r_B^e)}_{=p_A^{MH}}MH(r_A^e, r_B^e).$$

Similarly, the incremental value of advertising on platform B in addition to platform A is $\sigma(1 + \delta)\tau^*(r_B^e) - \sigma(\delta + SH_A(r_A^e, r_B^e) + MH(r_A^e, r_B^e)/2)\tau^*(r_A^e) = \sigma\tau^*(r_B^e)(SH_B(r_A^e, r_B^e) + MH(r_A^e, r_B^e)/2)$, again using the fact that $SH_A + SH_B + MH = 1$. Thus, platform B charges a price for multi-homers of $p_B^{MH} = \sigma\tau_B^{MH}(r_A^e, r_B^e)$, with $\tau_B^{MH}(r_A^e, r_B^e) = \tau^*(r_B^e)(SH_B(r_A^e, r_B^e) + MH(r_A^e, r_B^e)/2)$. Thus, the advertising price set by platform B is:

$$p_B = \underbrace{\sigma\tau_B^{SH}(r_A^e, r_B^e)}_{=p_B^{SH}} SH_B(r_A^e, r_B^e) + \underbrace{\sigma\tau_B^{MH}(r_A^e, r_B^e)}_{=p_B^{MH}} MH(r_A^e, r_B^e).$$

Using similar reasoning as in Lemma 2, we can show that the prices p_A and p_B given above constitute a unique equilibrium, where all advertisers multi-home on both platforms. So, the realized number of ads on each platform is equal to 1. Thus, consumers should rationally expect that $r_i = 1$, which excludes the case where $r_A^e < r_B^e$. As $r_A^e = r_B^e = 1$, we have $\theta\alpha(r_A^e) \leq \alpha(r_B^e)$ in equilibrium. Users spend an amount of time τ^* for each on-net communication. Single-homers of platform A spend a total amount of time communicating with all users $\tau_A^{SH} = \left(\delta + \hat{x}_A(1,1) + \theta(1 - \hat{x}_A(1,1))\right)\tau^*$ and single-homers of platform B spend $\tau_B^{SH} = \left(1 - \hat{x}_B(1,1) + \theta(\delta + \hat{x}_B(1,1))\right)\tau^*$. □

Proof of Lemma 6. For single-homers, we have

$$p_A^{SH} - p_B^{SH} = \frac{\sigma\tau^*\delta t(1-\theta)}{t - (1-\theta)\alpha},$$

which is positive, increasing in δ, and decreasing in θ. For multi-homers, we have

$$p_A^{MH} - p_B^{MH} = \frac{\sigma\tau^*\delta t}{t - (1-\theta)\alpha},$$

which is also positive, increasing in δ, and decreasing in θ. □

Proof of Lemma 7. The total demand of platform A on the competitive segment is equal to $\hat{x}_A$ and the total demand of platform B is equal to $1 - \hat{x}_B$. We have $d\hat{x}_A/d\delta > 0$ and $d\hat{x}_B/d\delta > 0$. We also have $dSH_A/d\delta > 0$, $dSH_B/d\delta < 0$ and $dMH/d\delta > 0$. □

Proof of Lemma 8. Derivating Equations (13) and (14) with respect to the level of interoperability θ at $r_i^e = 1$ and $x_i^e = \hat{x}_i$, we obtain:

$$\frac{d\hat{x}_A}{d\theta} = \frac{\alpha}{t}\left[-(SH_A + \delta) + (1-\theta)\frac{d\hat{x}_B}{d\theta}\right],$$
$$\frac{d\hat{x}_B}{d\theta} = \frac{\alpha}{t}\left[SH_B + (1-\theta)\frac{d\hat{x}_A}{d\theta}\right].$$

Let e_i be the exclusive demand of platform i. So, we have $e_A = SH_A + \delta$ and $e_B = SH_B$, with $e_A - e_B = \delta t/(t - \alpha(1-\theta)) > 0$. Solving the system of equations above for $d\hat{x}_A/d\theta$ and $d\hat{x}_B/d\theta$, we obtain:

$$\frac{d\hat{x}_A}{d\theta} = \frac{-\alpha}{t^2 - \alpha^2(1-\theta)^2}\left[te_A - \alpha(1-\theta)e_B\right],$$
$$\frac{d\hat{x}_B}{d\theta} = \frac{\alpha}{t^2 - \alpha^2(1-\theta)^2}\left[te_B - \alpha(1-\theta)e_A\right].$$

It follows that:

$$\frac{dMH}{d\theta} = \frac{d\hat{x}_A}{d\theta} - \frac{d\hat{x}_B}{d\theta} = -\frac{(e_A + e_B)\alpha}{t + \alpha(1-\theta)} < 0,$$

which proves the first point of the lemma.

Besides, since $t > \alpha(1-\theta)$ from Assumption 1 and $e_A > e_B$, then $d\hat{x}_A/d\theta < 0$, which implies that $dSH_B/d\theta > 0$. By contrast, the sign of $d\hat{x}_B/d\theta$ is ambiguous. We find that:

$$\frac{dSH_A}{d\theta} = \frac{d\hat{x}_B}{d\theta} = \frac{\alpha}{2}\left[\frac{-\delta t}{[t - \alpha(1-\theta)]^2} + \frac{2(t - \eta v_0) + \delta t}{[t + \alpha(1-\theta)]^2}\right].$$

This expression is increasing in θ, thus $dSH_A/d\theta > 0$ if and only if $\theta > \theta^{sup}$, where

$$\theta^{sup} = \frac{-(t-\alpha)(t - \eta v_0) - \delta t^2 + t\sqrt{\delta t\left[2(t - \eta v_0) + \delta t\right]}}{(t - \eta v_0)\alpha}.$$

Besides, we have $\theta^{sup} > 0$ if and only $\delta > \hat{\delta}$, with

$$\hat{\delta} = \frac{(t-\alpha)^2(t - \eta v_0)}{2\alpha t^2} < \delta^{\max}.$$

To complete the proof, we define $\hat{\theta} = \max\{0, \theta^{sup}\}$. □

Proof of Proposition 3. The variations of platform B's profit with respect to the interoperability level θ are given by:

$$\begin{aligned}
&\frac{d\Pi_B}{d\theta}\cdot\frac{1}{\sigma\tau^*} \\
&= -\frac{d\hat{x}_A}{d\theta}\left[\theta(\hat{x}_B + \delta) + \hat{x}_A - \hat{x}_B\right] \\
&\quad - \frac{d\hat{x}_B}{d\theta}\left[1 - \hat{x}_B + (1-\theta)(1 - \hat{x}_A)\right] + (1 - \hat{x}_A)(\delta + \hat{x}_B) \\
&= -\frac{d\hat{x}_A}{d\theta}\left[\theta(\hat{x}_B + \delta) + \hat{x}_A - \hat{x}_B\right] \\
&\quad - \left(\frac{t}{\alpha}\right)(1 - \hat{x}_A)\frac{d\hat{x}_A}{d\theta} - \frac{d\hat{x}_B}{d\theta}(1 - \hat{x}_B) \\
&= -\frac{d\hat{x}_A}{d\theta}\big[\theta(\hat{x}_B + \delta) + \hat{x}_A - \hat{x}_B \\
&\quad + (t/\alpha)(1 - \hat{x}_A)\big] - \frac{d\hat{x}_B}{d\theta}(1 - \hat{x}_B),
\end{aligned}$$

where we have used (15) to simplify between the first and second lines. Now, using the expressions for $d\hat{x}_A/d\theta$ and $d\hat{x}_B/d\theta$ in the proof of Lemma 8, we have:

$$-\frac{d\hat{x}_A}{d\theta} - \frac{d\hat{x}_B}{d\theta} = \frac{\alpha(e_A - e_B)}{t - \alpha(1-\theta)} > 0.$$

as $e_A = \hat{x}_B + \delta > 1 - \hat{x}_A = e_B$. Moreover, we have $\theta(\hat{x}_B + \delta) + \hat{x}_A - \hat{x}_B + (t/\alpha)(1 - \hat{x}_A) > \theta(\hat{x}_B + \delta) + \hat{x}_A - \hat{x}_B + (1 - \hat{x}_A) = \theta(\hat{x}_B + \delta) + 1 - \hat{x}_B$ as $t/\alpha > 1$ under Assumption 1. Since $-d\hat{x}_A/d\theta > d\hat{x}_B/d\theta$ and $\theta(\hat{x}_B + \delta) + 1 - \hat{x}_B \geq 1 - \hat{x}_B > 0$, then $d\Pi_B/d\theta > 0$. So, platform B always prefers perfect interoperability ($\theta_B^\star = 1$).

We now turn to the variations of platform A's profit with respect to θ. First, we show that the profit function Π_A is convex in θ. Indeed, $d^2\Pi_A/d\theta^2$ can be written as the sum of three terms, $d^2\Pi_A/d\theta^2 = t_0 + t_1 + t_2$, by collecting the terms in δ^0, δ^1 and δ^2, each of which is positive. First, we have

$$t_0 = \frac{2\alpha(t - \eta v_0)^2(2t - \alpha(1-\theta))\sigma\tau^*}{[t + \alpha(1-\theta)]^4} \geq 0.$$

The second term can be written as:

$$\begin{aligned}
t_1 = \delta\sigma\tau^* &\cdot \frac{2t\alpha}{(t + \alpha(1-\theta))(t^2 - (1-\theta)^2\alpha^2)^3} \\
&\cdot \big[(t - \eta v_0)(t - \alpha(1-\theta))^3(2t - \alpha(1-\theta)) \\
&+ \eta v_0\alpha(t + \alpha(1-\theta))(t^2 + 3\alpha^2(1-\theta)^2) \\
&+ \alpha^2(1-\theta)(t + \alpha(1-\theta))(3t^2 + \alpha^2(1-\theta)^2)\big],
\end{aligned}$$

which is positive. Finally, the third term is

$$\begin{aligned}
t_2 &= \frac{\sigma t\alpha\delta^2\tau^*}{2}\left[\frac{3t^2}{[t - \alpha(1-\theta)]^4} - \frac{t - 3\alpha}{[t - \alpha(1-\theta)]^3}\right. \\
&\quad \left. + \frac{3t^2}{[t + \alpha(1-\theta)]^4} - \frac{t + \alpha}{[t + \alpha(1-\theta)]^3}\right] \\
&= \frac{\sigma t\alpha\delta^2\tau^*}{2}\left[\frac{3t^2 - (t - 3\alpha)(t - \alpha(1-\theta))}{[t - \alpha(1-\theta)]^4}\right. \\
&\quad \left. + \frac{2t^2 - (2-\theta)\alpha t - (1-\theta)\alpha^2}{[t + \alpha(1-\theta)]^4}\right],
\end{aligned}$$

where we have simplified the first two terms and the last two terms in brackets in the first line to get the second line. The nominator of the first term in brackets in the second line is greater than the nominator of the second term, as the difference is equal to $2\alpha(2t+(t-\alpha)(1-\theta))$, which is positive under Assumption 1, while the denominator of the first term is lower than the denominator of the second term. So, we conclude that $t_2 \geq 0$.

Since Π_A is convex in θ, it is maximized at either $\theta = 0$ or $\theta = 1$. Let us define $\Delta_{\Pi_A} \equiv \Pi_A(\theta=1) - \Pi_A(\theta=0)$, which can be written as:

$$\begin{aligned}\frac{\Delta_{\Pi_A}}{\sigma\tau^*} &= -\delta^2 \frac{2\alpha t(2t^2-\alpha^2)+\alpha^2(t^2-\alpha^2)}{2(t^2-\alpha^2)^2} \\ &- \delta \frac{2t^3(\eta v_0+\alpha)-(t^2-\alpha^2)(t(t+\eta v_0)+\alpha\eta v_0)}{t(t^2-\alpha^2)(t+\alpha)} \\ &+ \frac{(t-\eta v_0)^2}{(t+\alpha)^2}.\end{aligned}$$

First, we have
$$\Delta_A\big|_{\delta=0} = \frac{(t-\eta v_0)^2\sigma\tau^*}{(t+\alpha)^2} > 0.$$

Second, $\partial\Delta_A/\partial\delta$ has the sign of a function $f(t,\alpha,\delta,\eta v_0)$. We find that $\partial f/\partial\delta = -kt^5[4+k(1-k(2+k))] < 0$, where $k \equiv \alpha/t \in (0,1)$. Hence, f decreases with δ. Besides, $f|_{\delta=0}$ can be either positive or negative. If $f|_{\delta=0} < 0$, f is always negative and thus, Δ_A decreases with δ. If $f|_{\delta=0} > 0$, then f is first positive then negative, so Δ_A first increases then decreases with δ. Since $\Delta_A\big|_{\delta=0} > 0$ and $\Delta_A < 0$ when δ goes to infinity, there is a threshold δ_{Π_A} such that $\Delta_A > 0$ for $\delta < \delta_{\Pi_A}$ and $\Delta_A < 0$ for $\delta > \delta_{\Pi_A}$. Thus, platform A would like to set $\theta_A^\star = 1$ if $\delta < \delta_{\Pi_A}$, and $\theta_A^\star = 0$ otherwise.

Defining $y = (t-\eta v_0)/t$ and $z = \alpha/t$, we find that:

$$\delta_{\Pi_A} = \frac{\begin{aligned}&y\left(1-2z+2z^2-z^4\right) \\ &-z(1-z)(1+z)^2+z(1-z^2)(1+z) \\ &\sqrt{1-\tfrac{2y(1-z+z^2+z^3)}{z(1+z)^2}+\tfrac{y^2(1+4z-4z^2+z^4)}{z^2(1+z)^2}}\end{aligned}}{4z-z^4-2z^3+z^2}.$$

Note that $y \in (0,1/2)$ from Assumption 3 and $z \in (0,1)$ from Assumption 1. We can then plot $\delta_{\Pi_A}(y,z)$ and $\delta^{\max}(z) = (1-z)/(2z)$, and we find that $\delta_{\Pi_A}(y,z) \leq \delta^{\max}(z)$ for all $y \in (0,1/2)$ and $z \in (0,1)$. □

Proof of Lemma 9. We find that consumer surplus is convex with respect to the level of interoperability θ:

$$\frac{d^2CS}{d\theta^2} = \frac{3t\alpha^2}{2}\left[\frac{\delta^2t^2}{(t-\alpha(1-\theta))^4} + \frac{\left(\delta t+2(t-\eta v_0)\right)^2}{(t+\alpha(1-\theta))^4}\right] > 0. \tag{A1}$$

Therefore, CS is maximized at either $\theta = 0$ or $\theta = 1$. Denoting $\Delta_{CS} \equiv CS(\theta=1) - CS(\theta=0)$, we have:

$$\begin{aligned}\Delta_{CS} &= -\delta^2 \cdot \frac{\alpha^2t(3t^2-\alpha^2)}{2(t^2-\alpha^2)^2} + \delta\cdot\frac{\alpha(2t+\alpha)(t-\eta v_0)}{(t+\alpha)^2} \\ &+ \frac{\alpha(2t+\alpha)(t-\eta v_0)^2}{t(t+\alpha)^2}.\end{aligned}$$

Making the change of notations $y = (t-\eta v_0)/t$ and $z = \alpha/t$, we find that $\Delta_{CS} < 0$ if and only if $\delta > \delta_{CS}$ with

$$\delta_{CS} = \frac{2-3z+z^3+\left(1-z^2\right)\sqrt{4-z^2}}{(3-z^2)z}y.$$

This threshold can be lower or higher than the maximum allowed value for δ, $\delta^{\max}$, defined after Assumption 3. Specifically, we have $\delta_{CS}(y,z) < \delta^{\max}(z)$ if and only

$$y < \hat{y}(z) = \frac{z(1+z)-2+(1+z)\sqrt{4-z^2}}{4z(1+z)},$$

with $\hat{y}(z) \in (0,1/2)$ for $z \in [0,1]$. □

Proof of Proposition 4. We find that $\delta_{CS} > \delta_{\Pi_A}$ for all $y \in (0,1/2)$ and $z \in (0,1)$. For example, this can be seen in the 3D plot of $\delta_{CS}(y,z) - \delta_{\Pi_A}(y,z)$. Therefore, for $\delta < \delta_{\Pi_A}$ or $\delta > \delta_{CS}$, the equilibrium level of interoperability coincides with the consumers' preferred level of interoperability. However, for $\delta_{\Pi_A} < \delta < \delta_{CS}$, it is too low from a consumer perspective. □

Proof of Proposition 5. Total welfare is defined as the sum of platforms' profits and consumer surplus, that is, $W = \Pi_A + \Pi_B + CS$ (the surplus of advertisers is zero). First, we show that the sum of platforms' profits is convex in θ. Indeed, we have:

$$\begin{aligned}&\frac{d^2(\Pi_A+\Pi_B)}{d\theta^2} \\ &= 2\sigma\tau^*\Bigg[\frac{2\delta^2\alpha t^3\left[t^4+8\alpha^2t^2(1-\theta)^2+3\alpha^4(1-\theta)^4\right]}{(t^2-(1-\theta)^2\alpha^2)^4} \\ &+ \frac{2\delta t\alpha(t-\eta v_0)(2t-\alpha(1-\theta))}{(t+(1-\theta)\alpha)^4} \\ &+ \frac{2(t-\eta v_0)^2\alpha(2t-\alpha(1-\theta))}{(t+\alpha(1-\theta))^4}\Bigg] \geq 0.\end{aligned}$$

Since CS is also convex in θ, then W is convex in θ, and it is maximized at either $\theta = 0$ or $\theta = 1$. Let $\Delta_\Pi = (\Pi_A+\Pi_B)(\theta=1) - (\Pi_A+\Pi_B)(\theta=0)$, we find that $\Delta_\Pi = \sigma\tau^*\overline{\Delta}_\Pi$, with:

$$\overline{\Delta}_\Pi = \frac{-2[\delta+y(1-z)]\times[\delta z-y(1-z)]}{\left(1-z^2\right)^2}.$$

where we have made the change of notations $y = (t-\eta v_0)/t$ and $z = \alpha/t$. We find that $\overline{\Delta}_\Pi > 0$ if $\delta < \delta_\Pi \equiv \frac{y(1-z)}{z} \leq \frac{1}{2}\frac{(1-z)}{z} = \delta^{\max}$, as $y \in (0,1/2)$. Besides, we find that $\delta_{\Pi_A} < \delta_\Pi < \delta_{CS}$. Therefore, there exists a threshold $\delta_W \in (\delta_\Pi, \delta_{CS})$ such that W is maximized as $\theta = 1$ if $\delta < \delta_W$, and at $\theta = 0$ otherwise. □

Appendix B

Equilibria in the Baseline Case

In this appendix, we show that under Assumptions 1 and 2, there is a unique equilibrium with market-sharing and partial multi-homing.

i. **Equilibria with market-sharing and partial multi-homing**
A market-sharing equilibrium with both single-homers and multi-homers exists if $0 < \hat{x}_B < \hat{x}_A < 1$, where

$$\hat{x}_B = \frac{t-\eta v_0}{t+(1-\theta)\alpha} \quad \text{and} \quad \hat{x}_A = 1 - \frac{t-\eta v_0}{t+(1-\theta)\alpha}.$$

We have $\hat{x}_B > 0$ and $\hat{x}_A < 1$ if and only if $\eta v_0 < t$. Then, we have $\hat{x}_B < \hat{x}_A$ if and only if $\eta v_0 > t/2 - (1-\theta)(\alpha/2)$, which is true for all $\theta \in [0,1]$ if and only if $\eta v_0 > t/2$. Thus, we have $0 < \hat{x}_B < \hat{x}_A < 1$ if and only if Assumption 2 holds. Under Assumptions 1 and 2, this is the unique equilibrium with market-sharing and partial multi-homing.

ii. Other possible equilibria
Under Assumptions 1 and 2, there are no other equilibria. First, there is no equilibrium with no adoption. A consumer located at y who expects no other users to join would receive the net utility $v_0 - t|y-y_i|$, with $y_i \in \{0,1\}$. However, since $v_0 > 0$, consumers close to A or B would derive a positive utility from joining and would thus join. Therefore, consumers should expect at least some of them to join.

Second, there are no equilibria with tipping. Assume, without loss of generality, that the market tips towards A, meaning that all the single-homers join A. Thus, we have $\hat{x}_A = 1$. Using (6), when $x_A^e = \hat{x}_A = 1$, $\hat{x}_B$ satisfies:
$$\hat{x}_B = 1 - \frac{\eta v_0}{t},$$

which is positive and less than 1 under Assumption 2. Thus, although A captures all the single-homers, B obtains some users through multi-homing. This is an equilibrium if, when consumers expect $x_B^e = \hat{x}_B = 1 - \frac{\eta v_0}{t}$, the realization of $\hat{x}_A$, given by (5) for $r_A^e = r_B^e = 1$, is such that

$$\frac{\eta v_0 + (1-\theta)\alpha \hat{x}_B}{t} \geq 1.$$

that is,

$$\frac{\eta v_0}{t} + (1-\theta)\frac{\alpha}{t}\left(1 - \frac{\eta v_0}{t}\right) \geq 1.$$

Since $\alpha < t$ under Assumption 1, $\theta \in (0,1)$, and $\eta v_0/t \in (1/2, 1)$ under Assumption 2, this condition is never satisfied. So, there are no equilibria with tipping under Assumptions 1 and 2.

Appendix C

Equilibria With Asymmetric Platforms

In this appendix, we show that under Assumptions 1 and 3, there is a unique equilibrium with market-sharing and partial multi-homing when the platforms are asymmetric.

i. **Equilibria with market-sharing and partial multi-homing**
A market-sharing equilibrium with both single-homers and multi-homers exists if $0 < \hat{x}_B < \hat{x}_A < 1$, where

$$\hat{x}_A = \frac{\eta v_0 + \alpha(1-\theta)}{t + (1-\theta)\alpha} + \delta \frac{(1-\theta)\alpha t}{t^2 - (1-\theta)^2\alpha^2} \quad \text{and}$$
$$\hat{x}_B = \frac{t - \eta v_0}{t + (1-\theta)\alpha} + \delta \frac{(1-\theta)^2\alpha^2}{t^2 - (1-\theta)^2\alpha^2}.$$

We have $\hat{x}_B < \hat{x}_A$ if and only if $\eta v_0 > (t - (1-\theta)\alpha(1+\delta))/2$, which holds for all $\theta \in [0,1]$ if $\eta v_0 > t/2$. In addition, we have $\hat{x}_B > 0$ if and only if

$$\frac{\eta v_0}{t} < 1 + \frac{\delta \alpha^2 (1-\theta)^2/t}{t - \alpha(1-\theta)},$$

and $\hat{x}_A < 1$ if and only if

$$\frac{\eta v_0}{t} < 1 - \frac{\delta \alpha(1-\theta)}{t - \alpha(1-\theta)}. \tag{C1}$$

The more stringent condition is the latter one. Since the right-hand side of this condition increases with θ, it holds for all $\theta \in [0,1]$ if it holds for $\theta = 0$. Therefore, to obtain an equilibrium with market-sharing and partial multi-homing for all $\theta \in [0,1]$, we assume that:

$$\frac{1}{2} < \frac{\eta v_0}{t} < 1 - \frac{\delta \alpha}{t - \alpha},$$

which corresponds to Assumption 3.

ii. **Other possible equilibria**
Under Assumptions 1 and 3, there are no other equilibria. First, for the same reasons as in the baseline case, there is no equilibrium with no adoption (see Appendix B).

Furthermore, there is no equilibrium with tipping towards A, that is, where A wins all the single-homers in the competitive segment. To see that, assume that A wins all the single-homers, that is, $\hat{x}_A = 1$. Using (14) for $x_A^e = \hat{x}_A = 1$, $\hat{x}_B$ satisfies:

$$\hat{x}_B = 1 - \frac{\eta v_0}{t},$$

which is positive and less than 1 under Assumption 3. Thus, B obtains some users through multi-homing. This is an equilibrium if, when consumers expect $x_B^e = \hat{x}_B = 1 - \frac{\eta v_0}{t}$, the realization of $\hat{x}_A$, given by (13) for $r_A^e = r_B^e = 1$, is such that

$$\frac{\eta v_0 + (1-\theta)\alpha(\hat{x}_B + \delta)}{t} \geq 1.$$

that is,

$$\frac{\eta v_0}{t} \geq 1 - \frac{\delta \alpha(1-\theta)}{t - \alpha(1-\theta)},$$

which does not hold under Assumption 3, as this assumption implies that condition (C1) holds (see above). So, there are no equilibria with tipping towards A under Assumptions 1 and 3. Similarly, we can prove that under Assumptions 1 and 3, there is no equilibrium where B wins all the single-homers, that is, $\hat{x}_B = 0$. Using the same approach as above, this would require that

$$\frac{\eta v_0}{t} \geq 1 + \frac{\delta\left(\frac{\alpha(1-\theta)}{t}\right)^2}{1 - \alpha(1-\theta)/t},$$

which cannot hold under these assumptions.

Appendix D

Effects of Interoperability on Profits Under Platform Asymmetry

In this appendix, we determine how platform asymmetry affects the three key effects of interoperability on platform profits.

Market power effect

The market power effect for platform A and platform B is given by:

$$\begin{aligned} mp_A &= \left(p_A^{SH} - p_A^{MH}\right)\frac{d\hat{x}_B}{d\theta} \\ &= \left[\theta(1-\hat{x}_A) + \frac{\hat{x}_A - \hat{x}_A}{2}\right]\frac{d\hat{x}_B}{d\theta} \\ mp_B &= \left(p_B^{SH} - p_B^{MH}\right)\left(-\frac{d\hat{x}_A}{d\theta}\right) \\ &= \left[\theta(\hat{x}_B + \delta) + \frac{\hat{x}_A - \hat{x}_A}{2}\right]\left(-\frac{d\hat{x}_A}{d\theta}\right). \end{aligned}$$

Let $\Delta p_i \equiv p_i^{SH} - p_i^{MH}$, for $i = A, B$, denote the premium that platform i can charge for its exclusive single-homers relative to (non-exclusive) multi-homers. We find that $\Delta p_B - \Delta p_A = \theta\left(\hat{x}_A + \hat{x}_B + \delta - 1\right) = \frac{\delta t}{t - \alpha(1-\theta)} > 0$. We also have $\frac{-d\hat{x}_A}{d\theta} > \frac{d\hat{x}_B}{d\theta}$ (see the proof of Proposition 3). It follows that $mp_B > mp_A$.

Total viewership effect

The total viewership effect for platform A and platform B is given by:

$$\begin{aligned} tv_A &= p_A^{MH}\frac{d\hat{x}_A}{d\theta} = \left[\delta + \frac{\hat{x}_A + \hat{x}_A}{2}\right]\frac{d\hat{x}_A}{d\theta} \\ tv_B &= p_B^{MH}\left(-\frac{d\hat{x}_B}{d\theta}\right) = \left[1 - \frac{\hat{x}_A + \hat{x}_A}{2}\right]\left(-\frac{d\hat{x}_B}{d\theta}\right). \end{aligned}$$

We have $p_A^{MH} = \delta + \frac{\hat{x}_A + \hat{x}_A}{2} > 1 - \frac{\hat{x}_A + \hat{x}_A}{2} = p_B^{MH}$ and $\frac{d\hat{x}_A}{d\theta} < -\frac{d\hat{x}_B}{d\theta}$. Therefore, $tv_A < tv_B$, with $tv_A < 0$. So, the negative total viewership effect is larger (in magnitude) for platform A than for platform B.

Usage intensification effect

The usage intensification effect for platform A and platform B is given by:

$$\begin{aligned} ui_A &= \frac{dp_A^{SH}}{d\theta}(\hat{x}_B + \delta) + \frac{dp_A^{MH}}{d\theta}(\hat{x}_A - \hat{x}_B) \\ ui_B &= \frac{dp_B^{SH}}{d\theta}(1 - \hat{x}_A) + \frac{dp_B^{MH}}{d\theta}(\hat{x}_A - \hat{x}_B). \end{aligned}$$

First, we have $\frac{dp_B^{MH}}{d\theta} > 0 > \frac{dp_A^{MH}}{d\theta}$, so the usage intensification effect for multi-homers is greater for the small than for the large platform. Second, we have

$$\frac{dp_A^{SH}}{d\theta}(\hat{x}_B + \delta) - \frac{dp_B^{SH}}{d\theta}(1 - \hat{x}_A)$$
$$= -\frac{\delta\alpha t^2(1-\theta)\left[2(t-\eta v_0)+\delta t\right]}{[t-\alpha(1-\theta)]^2[t+\alpha(1-\theta)]^2} < 0,$$

so the usage intensification effect for single-homers is also greater for the small than for the large platform.