Industrial Organization 02

Oligopoly

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Outline

- Definition and examples
- Cournot model, where firms compete in quantities
- Bertrand model, where firms compete in prices
- Bertrand paradox
- From Bertrand to Cournot: capacity constraints
- Cournot competition with n firms
- Comparison of market power: monopoly, Cournot, and Bertrand

Introduction

Definition of an oligopoly

An industry or a market in which a small number of firms compete

- Most markets fit this description: telecommunications, software industry, but also mineral water industry, etc.
- In an oligopolistic market, a firm cannot ignore the behavior of its competitors...
- ... and the reaction of competitors to its own decisions
- The **theory of oligopoly** aims at studying these **strategic interactions**

Note: oligopoly with 2 firms = duopoly

An example

The oligopoly of mineral water producers in France:

- Three firms (Danone, Nestlé, and Neptune) hold about 80% of the market
- Products marketed under different brand names (**Danone**: Evian, Volvic, ... ; **Nestlé**: Perrier, Contrex, ... ; **Neptune** (formerly Castel): Saint-Yorre, Vichy, Cristalline, ...) → *multiproduct firms*
- Creation of the Castel group in 1993 following a decision by the European Commission authorizing the acquisition of *la Société des eaux minérales du bassin de Vichy*
- Differentiation between mineral waters (health, well-being, different types of packaging) makes it difficult to compare prices
- Interdependence of strategic decisions: if one firm increases the price of mineral water, its competitors can choose to do the same or not react, hoping to capture a part of the customer base

Cournot model

- Cournot model (1838), developed by Augustin Cournot, a French engineer
- Two firms produce identical (homogeneous) goods ("perfect substitutes") and compete in quantities
- The price is set so that the total quantity of goods produced is sold; hence, the market price is $p = P(Q)$, where $Q = q_1 + q_2$ is the total quantity produced
- The marginal cost of producing each unit of the good is assumed to be constant and identical for both firms: c
- The profit function of firm $i \in \{1, 2\}$ is then

$$\Pi_i = (P(Q) - c)q_i$$

- For simplicity, we assume that $P(Q) = 1 - Q$

Cournot model

- Each firm i sets its quantity q_i to maximize its profits, given the quantity q_j set by the other firm (so, q_j is considered *fixed*)
- The **first-order condition** of the maximization problem for firm i is

$$1 - (q_1 + q_2) - c - q_i = 0$$

- We are looking for a **symmetric equilibrium** such that $q_i = q$. So:

$$q^\star = \frac{1-c}{3}$$

and

$$\Pi^\star = \frac{(1-c)^2}{9}$$

Cournot model

The **equilibrium profit** is:

$$\Pi^{\star} = \frac{(1-c)^2}{9} > 0$$

In Cournot competition, firms make profits!

However, the Cournot model seems somewhat unrealistic:

- Few examples of markets where firms set quantities rather than prices
- We don't know well how the price is set in the market

A model where firms set prices instead of quantities? → Bertrand model

Bertrand model

- Bertrand model (1883), developed by Joseph Bertrand, a French engineer
- Two firms, 1 and 2, produce identical goods ("perfect substitutes") and compete in prices
- The demand function is $q = D(p)$
- The marginal cost of production is constant and identical for both firms: c
- We assume (not crucial) that the market is split evenly if firms offer the same price. So, we have:

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}D(p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

- The profit function of firm i is:

$$\Pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j)$$

Bertrand equilibrium

We look for the Nash equilibrium for this one-stage game

Bertrand paradox

The game has a unique Nash equilibrium such that firms set $p_1^* = p_2^* = c$. At the equilibrium, we have $\Pi_1^* = \Pi_2^* = 0$ and $W = W^*$ (social welfare is maximized)

A **strong result**:

- As the number of firms increases from one (monopoly) to two (duopoly), the price falls from the monopoly price to the competitive price and stays at the same level as the number of firms continues to increase
- Two firms are sufficient to reach a perfectly competitive equilibrium
- Because this result is extreme and generally inconsistent with reality, it is called the "Bertrand paradox"

Proof

If $p_1^* > p_2^* > c$?

Then, firm 1 increases its profit by setting $p_1^* = p_2^* - \epsilon$

If $p_1^* = p_2^* > c$?

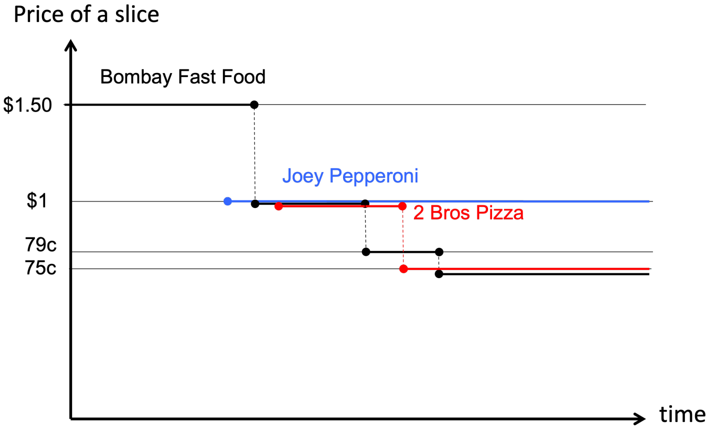
Then, firm 1 increases its profit by setting $p_1^* = p_2^* - \epsilon$, as for a small ϵ, we have:

$$D(p_1^*)(p_1^* - c)/2 < D(p_1^* - \epsilon)(p_1^* - c - \epsilon)$$

If $p_1^* > p_2^* = c$?

Then, firm 2 increases its profit by setting $p_2 = p_2^* + \epsilon$

An example: the pizza war in NYC



Why lower prices? *"He was taking away our customers"*.

Source: "In Manhattan Pizza War, Price of Slice Keeps Dropping", *New York Times*, March 2012.

Bertrand competition with different marginal costs

Let's assume that $c_1 < c_2$. We denote by $p^m(c)$ the monopoly price for a marginal cost of c

If costs are close enough: $c_1 < c_2 < p^m(c_1)$

- There is a unique Nash equilibrium with $p_1^* = c_2 - \epsilon$ and $p_2^* = c_2$
- Only firm 1 makes profit: $\Pi_1^* = (c_2 - c_1)D(c_2)$ and $\Pi_2^* = 0$

If firm 1 is much more efficient than firm 2: $c_1 < p^m(c_1) < c_2$

- There is a unique Nash equilibrium with $p_1^* = p^m(c_1)$ and $p_2^* = c_2$
- Only firm 1 makes profit: $\Pi_1^* = \Pi_1^m$ and $\Pi_2^* = 0$

Solutions to the Bertrand paradox

Four important solutions to the "Bertrand paradox", corresponding to four important assumptions on which the model is based:

1. Products are homogeneous
2. Short-run competition (a static analysis of a one-stage game)
3. Firms have no capacity constraints
4. Consumers are perfectly informed

Relaxing one of these assumptions solves the paradox:

1. Product differentiation
2. Dynamic competition (repeated interactions)
3. Firms are capacity constrained
4. Imperfect information

Product differentiation

For example, let's assume a **geographic differentiation**

- Two ice cream vendors, 1 and 2, are located at both ends of a beach
- If $p_1 = c$, is $p_2 = c + \epsilon > c$ possible?
- Consumers close to vendor 2 can prefer to buy a little more expensive from 2 than moving to 1!
- Horizontal and vertical differentiation theory: the Hotelling model

In the presence of product differentiation...

A situation such that $p_i > c$ can be an equilibrium

→ See the lecture on **differentiation**

Dynamic competition

- The Bertrand model assumes that firms compete only during a single period
- In this situation, as we start from $p_1 = p_2 > c$, a firm has strong incentives to undercut its price
- In a more dynamic framework, what can happen?
- In a dynamic situation, firms are going to take account the effect of their price cut on the rival's behavior in future periods
- If there is a "punishment" (price war...), firms will compare the short and long term gains

In a repeated interaction framework...

A situation where $p_i > c$ can be an equilibrium

→ See the lecture on **collusion**

Capacity constraints

- The Bertrand model assumes that firms do not have capacity constraints
- If $p_1 = p_2 = c$, the two firms share the demand: $D(c)/2$
- If firm 2 increases its price slightly, $p_2 = c + \epsilon$, we have assumed that firm1 meets the demand, that is, $D(c)$
- But firm 1 may not be able to satisfy all of the demand: capacity constraints
- In this case, firm 2 can increase its price and still keep some of the market

If firms face capacity constraints...

A situation where $p_i > c$ can be an equilibrium

$\rightarrow$ See below

Imperfect information

With imperfect information...

A situation with $p_i > c$ can be an equilibrium

The Diamond Paradox

- The same product is sold in many different stores
- But consumers are not informed about prices
- Cost ϵ to go from one store to another (*search cost*)
- If $p_1 = p_2 < p^m$, then deviation is possible to $p_1 + \epsilon/2$

Price competition with capacity constraints

We are going to study a model of price competition where firms have capacity constraints

We consider the **following model**:

- Two firms compete in prices, but are capacity constrained
- The demand function is linear, $D(p) = 1 - p$
- So, the inverse demand is: $p = P(q_1 + q_2) = 1 - (q_1 + q_2)$
- Firm i cannot produce more than its production capacity $\overline{q}_i$. So, we must have $q_i \leq \overline{q}_i$
- The unit cost of acquiring capacity $\overline{q}_i$ is $c_0 \in [3/4, 1]$
- There are no production costs ($c = 0$)

Price competition with capacity constraints

- We need to define a rationing rule
- The rationing rule determines which consumers will be served if the firm cannot meet all demand
- We use the "efficient" rationing rule: *consumers with higher willingness to pay are served first*
- If $p_1 < p_2$ and $\overline{q}_1 < D(p_1)$:

$$\widetilde{D}_2(p_2) = \begin{cases} D(p_2) - \overline{q}_1 & \text{if} \quad D(p_2) \geq \overline{q}_1 \\ 0 & \text{if} \quad D(p_2) < \overline{q}_1 \end{cases}$$

- Another possible rule: the "proportional" rationing rule

Results

We obtain the following result:

Price competition with capacity constraints

The unique Nash equilibrium is such that the firms set the same price:

$$p^* = 1 - \left(\overline{q}_1 + \overline{q}_2\right)$$

Consequence: at the stage of defining their production capacities $\overline{q}_i$, firms have a gross profit equal to:

$$\Pi_i^g = \left[1 - \left(\overline{q}_i + \overline{q}_j\right)\right]\overline{q}_i$$

That is? **The profit function in the Cournot model**

Proof: preliminaries

First, given that $c_0 \in [3/4, 1]$, what is the upper bound for $\overline{q}_i$?

To answer the question, what is the maximum profit for firm i?

$\rightarrow$ It is the monopoly profit!

As $D(p) = 1 - p$ and $c = 0$, the monopoly price is equal to...? $1/2$

So the monopoly profit is...? $1/4 - c_0\overline{q}_i$

Therefore, $\overline{q}_i \leq 1/3$ because we must have $1/4 - c_0\overline{q}_i \geq 0$

Proof of existence

Now, let's show that $p^* = 1 - (\bar{q}_1 + \bar{q}_2)$ is a Nash equilibrium

Observation: $p^* > c$ because $\bar{q}_1 + \bar{q}_2 < 2/3$

Can firm i lower her price?

No, she would not increase her profit as she is already at the maximum of her production capacity

→ "Bertrand competition" does not work because firms face capacity constraints

Proof of existence

If the firm i sets a higher price $p > p^*$?

- Firm j captures the entire demand... up to the maximum of its production capacity
- A fraction of the demand is not satisfied: we call it the residual demand
- The residual demand is equal to $1 - p - \bar{q}_j$
- So, firm i makes profit:

$$p\left(1 - p - \bar{q}_j\right) = \left(1 - q - \bar{q}_j\right)q$$

- It is the Cournot profit! It is concave in q and its derivative in $q = \bar{q}_i$ is

$$1 - 2\bar{q}_i - \bar{q}_j \geq 0 \text{ as } \bar{q}_i \leq 1/3$$

So, the optimal value of q is $q = \bar{q}_i$

Proof of uniqueness

$p^* = 1 - \left(\bar{q}_1 + \bar{q}_2\right)$ is the unique Nash equilibrium.

- $p_1 = p_2 = p > P\left(\bar{q}_1 + \bar{q}_2\right)$ is not an equilibrium because at least one firm does not reach its maximum production capacity. So, it can lower its price
- $p_1 = p_2 = p < P\left(\bar{q}_1 + \bar{q}_2\right)$ is not an equilibrium. With $p_i = p_i + \epsilon$, the firm would sell the same capacity (its production capacity) at a higher price
- $p_1 < p_2$ not feasible, because firm 1 is encouraged to increase its price

General result

Consider the following two-stage game:

- First, firms simultaneously choose their production capacity
- Second, firms observe the chosen capacities and simultaneously choose their prices

Kreps and Scheinkman (1983): capacity + price = quantities

If the demand function is concave and we use the efficient rationing rule, then the equilibrium of this two-stage game is equivalent to the equilibrium of Cournot competition (competition in quantities)

Cournot or Bertrand competition?

In a given situation, **which model is most appropriate**, Bertrand or Cournot?

Rule of thumb

If the production capacity can be easily adjusted, the Bertrand model is a better representation of duopoly competition. Otherwise, if it is difficult to adjust the production capacity, the Cournot model is more appropriate

Examples of markets where production capacity is difficult to adjust? → markets for physical goods (car manufacturers, airplanes, cement industry...)

Examples of markets where production capacity is easy to adjust: → service markets (banking, insurance...)

Content industries: Bertrand or Cournot?

In the **content industries** (music, movies, video games...), we have observed an evolution of the distribution model:

- *old distribution model*: sale of content on physical devices (CD, DVD, cartdridges, etc.)
- *new distribution model*: sale of digital content through online stores like iTunes or Steam

Which model of competition (Cournot, Bertrand) better represents the old distribution model? The new one?

What consequences can we predict from this change in the competition model?

Cournot competition with n firms

- Consider a market with the linear demand $D(p) = 1 - p$
- n firms operate in this market and produce a homogeneous good
- They compete in **quantities** (Cournot competition)
- The cost fonction of firm i is $C_i(q_i) = cq_i$ (constant marginal cost c)
- What is the equilibrium price?
- What is the equilibrium profit?

Cournot competition with n firms

- The inverse demand is $P(Q) = 1 - Q$, where

$$Q = \sum_{i=1}^{n} q_i$$

- The profit function of a firm i is $\Pi_i = (P(Q) - c)q_i$
- Firm i chooses its quantity q_i to maximize its profit
- The first order condition of the maximization problem is:

$$1 - Q - c - q_i = 0$$

- We look for a symmetric equilibrium with $q_i = q$. We find

$$q^\star = \frac{1-c}{n+1} \quad \text{and} \quad \Pi^\star = \frac{(1-c)^2}{(n+1)^2}$$

Cournot competition with n firms

The **equilibrium profit** is:

$$\Pi^{\star} = \frac{(1-c)^2}{(n+1)^2}$$

The greater the number of firms are, the lower the Cournot profit is

When the number of firms is large, firms make little profit

Comparison in terms of market power

Assumptions:

- Let's assume there are n firms
- They have the same marginal cost of production, c
- We define $L_i = (p_i - c)/p_i$ the Lerner index for firm i

Comparison: monopoly, Bertrand, Cournot

- In a monopoly, we have?

$$L_i = \frac{1}{\epsilon}$$

- In Bertrand competition, we have? $\rightarrow L_i = 0$
- In Cournot competition, we have?

$$L_i = \frac{\alpha_i}{\epsilon}, \text{ where } \alpha_i \text{ is the market share of firm } i$$

Proof

- Let $P(Q)$ represent the inverse demand function, where $Q = q_1 + q_2$ is the total quantity produced
- Then, firm i's profit function is

$$\left(P\left(q_i + q_j\right) - c\right) q_i$$

- The first-order condition is:

$$P\left(q_i + q_j\right) - c + q_i P'\left(q_i + q_j\right) = 0$$

- So,

$$\begin{aligned} \frac{P-c}{P} &= -\frac{q_i P'}{P} = -\frac{q_i}{Q}\frac{P'Q}{P} \\ &= -\frac{q_i}{Q}\frac{Q}{PD'} \end{aligned}$$

i.e.,

$$L_i = \frac{\alpha_i}{\epsilon}$$

Merger in the French mobile telephony market

- In March 2015, the French mobile telephony market had a customer base of 71 million subscribers (according to ARCEP), that is, a penetration rate of 107%
- There are four main operators: Orange (35.6%), SFR (28%), Bouygues Télécom (14.2%) and Free (13.2%)
- Mobile virtual network operators (La Poste, etc.) have small market shares (8.7%)
- There have been rumors of mergers between Bouygues Télécom and Free
- We are going to build a simple competition model to study the incentives of Bouygues Télécom and Free to merge

Merger in the French mobile telephony market

- For simplicity, let's assume that there are 4 players in the mobile market
- We thus ignore the competitive pressure coming from MVNOs
- We assume that these 4 firms produce homogeneous (identical) goods and compete in quantities
- Let q_i be the quantity of firm i and $Q = q_1 + q_2 + q_3 + q_4$ the total quantity
- The demand function is $D(p) = 50 - p$.

Questions:

1. Determine the Nash equilibrium for a quantity competition game. Compute the quantity and profit in equilibrium for each firm
2. Assume that firms 3 and 4 merge (Bouygues and Free) to form a new firm, BF. Compute again the equilibrium, the quantity produced and the equilibrium profit for firms BF, 1 and 2
3. Are firms 3 and 4 better off if they merge? How do the profits of firms 1 and 2 evolve?

Merger in the French mobile telephony market

1. Determine the Nash equilibrium for a quantity competition game. Compute the quantity and profit in equilibrium for each firm.

- The quantity in equilibrium for a Cournot model with n firms is:

$$q^*(n) = \frac{50}{n+1}$$

- The Cournot profit with n firms is:

$$\Pi_i^*(n) = \frac{2500}{(n+1)^2}$$

- Therefore, for $n = 4$, we have: $q^*(n) = 50/5 = 10$ and $\Pi_i^* = 2500/25 = 100$

Merger in the French mobile telephony market

1. We assume that firms 3 and 4 merge (Bouygues and Free) to form a new firm, BF. Compute again the equilibrium, the quantity produced and equilibrium profit for firms BF, 1 and 2.

- We move from an oligopoly with 4 firms to an oligopoly with 3 firms
- Thus, for $n = 3$, we have: $q^*(n) = 50/4$ and $\Pi_i^* = 2500/16 = 156.25$

Merger in the French mobile telephony market

1. Are firms 3 and 4 better off if they merge? How do the profits of firms 1 and 2 evolve?

- Bouygues and Free are not well advised to merge because $156.25 < 100 * 2$: their total profit decreases!
- On the other hand, Orange and SFR are better off: $156.25 > 100$!

Merger in Cournot competition

- Let's consider a market with $n > 1$ firms
- The marginal cost is constant and identical for all firms: c
- Linear demand: $P(Q) = a - bQ$
- In equilibrium, a firm makes the following profit:

$$\Pi_i^*(n) = \frac{1}{b}\frac{(a-c)^2}{(n+1)^2}.$$

- If k firms merge, $n - k + 1$ firms will remain in the market
- So, a merger involving k firms is profitable if

$$\Pi_i^*(n-k+1) \geq k\Pi_i^*(n)$$

Merger in Cournot competition

In Cournot competition, a merger is profitable only if it involves more than 80% of the firms in the market

Merger in Bertrand competition

- Let's consider a market with $n > 1$ firms
- The marginal cost is constant and identical for all firms: c
- In equilibrium, we have $p_1^* = ... = p_n^* = c$ and profits are equal to 0
- If $k < n$ firms merge, how is the equilibrium changed?
- It is **unchanged**: the price remains equal to c and profits are still equal to 0

Merger in Bertrand competition

In Bertrand competition, a merger is profitable only if it involves all (100%) of the firms in the market

Conclusion on mergers

Merger decisions cannot be explained by incentives to reduce competition in the market alone

Other dimensions to explain mergers?

- Synergies: cost reductions...
- Becoming an industry leader (the competition model is changing: Stackelberg instead of Cournot)
- Portfolio strategy: product line extension, economies of scale in sales and marketing, bundling...

Take-aways

- Bertrand competition between identical firms leads to marginal-cost pricing ("Bertrand paradox")
- The Bertrand paradox is no longer verified if we relax one of the 4 main assumptions...
- If capacity constraints can be easily adjusted in the short run, firms are more likely to compete à la Bertrand. If capacity is fixed in the medium run, firms compete à la Cournot
- When n firms compete in Cournot, their profit is inversely proportional to the number of firms playing in the market
- A merger is not always profitable in Cournot competition; it must involve at least 80% of the firms in the market. In Bertrand competition, it requires 100% of the firms in the market. Mergers cannot be explained by an incentive to reduce competition alone